Equitable Distribution of Recharging Stations for Electric Vehicles

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June 6, 2017

Abstract

Given the limited driving range of battery electric vehicles and lack of sufficient charging infrastructure, locating charging stations is an important decision problem to enable long-distance travels by battery electric vehicles. This paper considers an important political factor in such location problems: the equitable access to charging stations among geographical regions. We propose three types of equity constraints to the flow refueling location model: two constraints based on travel demand and the other based on traffic flow. For solving the problem with flow equity constraints, we propose a multi-phase heuristic method. We test the proposed models and computational method in a real expressway network in Korea.

Keywords: electric vehicles; flow refueling location problem; demand equity; flow equity

1 Introduction

In the United States, the transportation sector comprises 28% of total greenhouse gas emissions. Light-duty vehicles, such as passenger cars and light trucks are responsible for 62% of total greenhouse gas emissions [32]. As a way to solve the greenhouse gas problem of the transportation sector, electric vehicles (EVs) have attracted public attention. The CO2 emission per mile of EVs, including emission during electricity production, is approximately 50% of the CO2 emission per mile of fossil fuel vehicles [25]. Also, the travel cost of EVs (2-3 cents per mile) is cheaper than the travel cost of fossil fuel vehicles (13 cents per mile) [18]. Due to the environmental and economic advantages of EVs, the demand for EVs is predicted to gradually increase. However, there are two barriers to the widespread adoption of EVs. One is the limited driving range of EVs per refueling, and the other is an insufficient refueling station infrastructure [2, 15, 28]. The driving range of EVs is typically much shorter than the driving range of fossil fuel vehicles. The driving range of most EVs on the market since 2014 is about 60 kilometers to 160 kilometers [5]. The only way for a mass

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number of drivers to utilize EVs as a primary vehicle, despite the limited driving range of EVs, is the construction of a sufficient refueling station infrastructure. However, no investors are interested in supporting an EV refueling station infrastructure under the circumstances that there are only a few EVs. On the other hand, consumers will not buy EVs if the refueling station infrastructure remains insufficient [22, 29]. To facilitate the widespread adoption of EVs while solving this early-stage problem, government support for constructing infrastructure is needed [6]. Moreover, the number of refueling stations that can be constructed with initial government support is not sufficient to cover all EV demand. Accordingly, strategic planning to construct refueling stations at optimal locations is necessary.

In this paper, we consider a *public* EV recharging station location problem, especially when the government is involved in the location decision and the operations of those stations. As illustrated in Section 4, we consider the Korean Expressway network as a case study subject, which is managed and operated by the government and public companies; therefore charging stations become public properties. There have been a lot of research works on how to optimize an EV refueling station network [13, 14, 31, 34]. The majority of previous studies have focused on identifying the best locations for refueling stations to maximize traffic flow. Based on this objective, most studies designate optimal locations for refueling stations in metropolitan or downtown areas, where traffic flow is high. Taking into account the *public* nature and the *equitable* distribution of facilities, however, it is inappropriate to densely locate refueling stations in only certain regions. In particular, when it comes to designating locations for public service facilities—the objective of which is not profit maximization—it is important to equitably distribute the benefits of facilities to all stakeholders. Equity is more greatly influenced by location changes and facility capacity than efficiency [21]. Accordingly, equity should be prioritized in addressing issues of facility location, particularly in the context of public service facilities. In terms of designating locations for EV refueling stations, however, there is no research considering the issue of equity.

While there have been equity considerations in the literature of other transportation and public facility areas, it remains rather unclear what type of equity constraints is the most appropriate form to be considered in the context of EV recharging station location problems. To fill this gap in the literature, this paper suggests and compares three kinds of equity constraints for a refueling station location model, and applies them to an arc-cover path-cover flow refueling location model (AC-PC FRLM) proposed by Capar et al. [3]. We compare three equity constraints in terms of both computational efficiency and locational differences.

The remainder of this paper is organized as follows. In Section 2, we review related research. The formulation of AC-PC FRLM and equity constraints are introduced in Section 3. In Section 4, numerical experiments are conducted using data from the Korean Expressway network. We discuss a way to generate equitable distribution networks and propose a multi-phase method to efficiently find solutions. Section 5 concludes the paper with suggestions for future research.

2 Literature Review

We provide a review of the current literature. We first introduce related research results in refueling/recharging station location problems for alternative-fuel or electric vehicles. Then we discuss equity issues considered in general other transportation problems.

2.1 Refueling Station Locations

Several attempts have been made to designate optimal locations for EV refueling stations. The existing work on the topic can be classified according to its perspective on demand, with node-based demand models and flow-based demand models. The p-median model is a well-known node-based demand model. The p-median model locates p facilities to potential nodes to minimize the total weighted distance between demand nodes and the nearest facilities [9]. Nicholas and Ogden [26] used the p-median model to determine the number of hydrogen refueling stations required to satisfy a certain level of service. Similarly, Lin et al. [17] determined optimal locations for hydrogen refueling stations using the p-median model. The authors used vehicle miles traveled to minimize the total fuel-travel-back travel time. However, node-based demand models are not suitable for the problem of determining locations for refueling stations, because people prefer to visit refueling stations on the way to a destination rather than to visit stations only for purposes of refueling [11]. Moreover, Upchurch and Kuby [30] compared the p-median model and FRLM by applying the locations derived from each model to the objective function of the other. The results showed that the location scheme derived from the FRLM performs better than that of the p-median model.

Flow-based demand models can be classified into sets of covering location models and maximal covering location models. Wang and Lin [34] developed a flow-based set covering model that is applicable when traffic flow data is not available. They applied this set covering model to the Taiwan network, and observed changes in the number of refueling stations according to changes in driving ranges and refueling coefficients representing the amount of fuel required for a vehicle to completely refuel at each node. Wang and Wang [36] proposed a multi-objective model that minimizes costs and maximizes covered population based on vehicle refueling logics, and solved it with a weighted sum method. Wang and Lin [35] extended the model by Wang and Lin [34], taking into account multi-type refueling stations, capacity of nodes, and budget limitations.

The second type of flow-based demand model is a maximal covering location model. Hodgson [10] introduced a maximal covering location model that locates p facilities to maximize the total traffic flow covered. The model is called the flow capturing location model (FCLM), and is the basis of FRLM. In FCLM, demand is expressed as the traffic flow between pairs of origin and destination (OD), and traffic flow is captured when there is at least one service facility located on the OD route. It is not appropriate to apply FCLM to the EV refueling station location problem, because an EV may need to be refueled more than once to complete a long-distance trip due to the limited driving range of EVs.

Kuby and Lim [14] extended FCLM to FRLM in order to account for the need for multiple refueling stops on a path by using combinations of nodes that enable round trips without fuel shortages. This model requires combinations of nodes for all OD pairs, which are determined in a pre-generation stage. Thus it is time-consuming work for any large-scale network. To avoid pre-generation work and reduce computation time, Lim and Kuby [16] developed three heuristic algorithms (a greedy-adding algorithm, a greedy-adding with substitution algorithm, and a genetic algorithm) that do not require the generation of combinations to solve FRLM. Applying the heuristic algorithms to the state of Florida's network obtained solutions in a way that typical FRLM was unable to, because the extensive generation of combinations was impossible for such a large-sized network. Capar and Kuby [2] developed a new formulation for FRLM that does not require the pre-generation of combinations. They also experimented with the formulation in the context of the Florida state network, comparing the new formulation with the heuristic algorithms [16]. MirHassani and Ebrazi [24] proposed the reformulation of FRLM with the concept of an expanded network.

Capar et al. [3] introduced AC-PC FRLM to determine an optimal location scheme for refueling stations. In AC-PC FRLM, a traffic flow is successfully refueled when all the arcs in the route are passable without an out-of-fuel status. The results of this experiment showed that AC-PC FRLM provides faster solutions than the formulation by Capar and Kuby [2] in large-scale networks. Our formulation and computation method are developed based on AC-PC FRLM.

Some researchers have extended the basic FRLM model in various directions. Upchurch et al. [31] extended FRLM by considering the capacity of refueling stations. Kim and Kuby [13] relaxed one of the assumptions of FRLM, which states that drivers use a fixed route. The authors took into account that drivers change their routes to accommodate nearby refueling stations, which are not always on the shortest or most direct route to a destination. Chung and Kwon [5] proposed multi-period FRLM based on the formulation of MirHassani and Ebrazi [24]. They compared the multi-period model with a single-period model that solves for each period based on the Korean Expressway network.

In this paper, we use the AC-PC FRLM model of Capar et al. [3] as the base model.

2.2 Equity

In making decisions regarding public services, it is important to take equity into account. The equity issue has been considered in several areas of research, including facility location problems for public services, hazardous materials management, and road network design. Marsh and Schilling [20] reviewed equity measures used in facility location models and developed a framework for equity measures. The authors also identified the characteristics of good equity measures to help determine proper equity measures for general application. Mandell [19] considered equity in public services using the Gini coefficient and developed a bi-criteria optimization model to maximize output (i.e., effectiveness) and minimize the Gini coefficient (i.e., equity). Batta et al. [1] developed a p-maxian facility location model with constraints for population, dispersion, and equity.

In the area of hazardous materials management, Gopalan et al. [8] developed an integer programming formulation for hazardous materials routing that constrains the gap of risks between any two regions. Current and Ratick [7] developed a multi-objective model that determines optimal facility location and routing in hazardous materials management. They considered equity so that the objective function minimizes maximum waste shipped to nodes and maximum waste shipped to facilities from nodes. Kang et al. [12] added equity constraints to a hazardous materials routing problem, and solved it with a Lagrangian relaxation method.

Road network design is also an important area of application where equity issues should be addressed. Chen and Yang [4] introduced two kinds of stochastic programming models that consider equity and demand uncertainty for network design problems—an expected-value model and a chance-constrained model. Ohsawa et al. [27] suggested a bi-criteria facility location model, which uses facility-inhabitant distances as an equity measure. Meng and Yang [23] applied equity to a continuous network design problem. As the costs for road networks in some regions are relatively higher than the costs in other regions, with the common problem of minimizing total costs, the authors developed an equity constrained bi-level programming model to limit cost increases by region, applying various scenarios. Wang et al. [33] developed a relaxation algorithm for an equity constrained bi-level programming model.

2.3 Our Contributions

While there are a number of options available to constraint the equity among entities, our focus in this paper is to investigate which measure of quality should be put equally. We suggest three types of equity constraints. Two of them are based on the travel demands originating from and heading to each region provided with recharging opportunities, and the other is based on the traffic flow volume passing through each region. Since equity constraints usually make the problem difficult to solve, by limiting the number of feasible solutions, we also face the same computational challenges. In this paper, we compare the two demand-based equity constraints for the EV recharging station location problem in terms of computational efficiency, while we propose a heuristic algorithm for the case with the flow-based equity constraint.

3 Model Formulation

This section introduces a base model adopted from Capar et al. [3], then propose three types of equity constraints.

3.1 The Arc-Cover Path-Cover Flow Refueling Location Model (AC-PC FRLM)

Some assumptions are established for the AC-PC FRLM:

1. An OD pair is considered to be refueled when a round trip is available;

2. A driver travels on the shortest path;

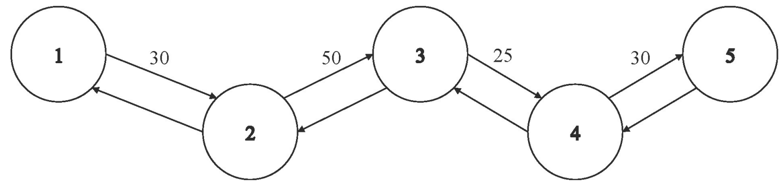


Figure 1: Simple network

3. A refueling station can be located in a node;
4. Fuel consumption is linearly proportional to distance traveled; and
5. The capacity of a refueling station is unlimited.

The fuel level at each node is decided by the location of the refueling station and the distance between nodes on a round trip path for an OD pair. When a vehicle is at a node designated as a refueling station, the fuel level at the node is full. Otherwise, the distance between a node and a nearby refueling station determines the fuel level. For example, let us consider the simple network in Figure 1, and assume that the driving range of a vehicle is a distance of 100 units. If a refueling station is located at node 1, then the fuel level at node 1 is 100 units. If a refueling station is located at node 2, then the driver travels from node 2 to 1 using 30 units of fuel, and the remaining fuel level at node 1 becomes 70 units.

With the notation summarized in Table 1, the AC-PC FRLM is expressed as:

$$\max_{y,z} \quad \sum_{q \in Q} f_q y_q, \tag{1}$$

$$\text{s.t.} \quad \sum_{i \in K_{jk}^q} z_i \geq y_q, \quad \forall q \in Q, a_{jk} \in A_q, \tag{2}$$

$$\sum_i z_i = p, \tag{3}$$

$$y_q, z_i \in \{0,1\}, \quad \forall q \in Q, i \in N. \tag{4}$$

Equation (1) is the objective function that maximizes the sum of refueled traffic volume. Equation (2) indicates that the traffic flow of the OD pair q is refueled when all arcs on the OD pair q are passable. The set of candidate nodes K_{jk}^q means that if at least one refueling station is located at the element K_{jk}^q, then the arc a_{jk} is passable. Let us consider the network in Figure 1, and again assume that the driving range is 100 units. Then for example, a driver can travel an arc, $a_{1,2}$, only if at least 30 units of fuel remain in the vehicle. If a refueling station is located at node 1 or node 2, then the remaining fuel is 100 or 70 units, respectively. If a refueling station is located at node 3, however, then the remaining fuel is 20 units, which means that the vehicle cannot travel $a_{1,2}$ because there is a lack of sufficient fuel. Therefore, nodes 1 and 2 become the candidate nodes for $a_{1,2}$ in this OD pair. In the case of a different arc, $a_{3,4}$, the fuel required is 25 units. If a refueling station is located at node 2, then the remaining fuel is 50 units, and node 2 becomes a

Table 1: Notation

Notation	Description
Indices	
i, j, k	Nodes
q	OD pairs
r	Regions
a_{jk}	Arc from node j to k
Sets	
N	Set of nodes
Q	Set of OD pairs
A_q	Set of arcs on OD pair q
K_{jk}^q	Set of candidate nodes where EV refuels and is enabled to pass an arc a_{jk} on OD pair q
R	Set of regions
N_r	Set of nodes in region r
Q_r	Set of OD pairs whose origin or destination belongs to a node in region r
$\overline{Q}_r$	Set of OD pairs whose shortest path passes through at least a node in region r
Parameters	
f_q	Traffic volume on OD pair q
p	Number of refueling stations to be located
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Equity tolerance levels
D_r	Demand in region r
Variables	
z_i	Takes a value of 1 if a refueling station is located at node i, 0 otherwise
y_q	Takes a value of 1 if the traffic flow on OD pair q is refueled, 0 otherwise

candidate node for $a_{3,4}$. In this way, the candidate nodes are determined for all arcs in the network. Constraint (3) indicates that p refueling stations must be located. Constraints (4) define the binary variables.

3.2 Demand Equity Constraints

One simple way to develop equity constraints is the comparison of an equity measure between every two regions [8, 12]. We develop two equity constraints: demand equity and flow equity constraints. The first equity constraint limits the number of refueling stations at each region proportional to the demand of the region, determined as a set of nodes, as shown in equation (5):

$$\frac{\sum_{i\in N_{r_1}} z_i}{D_{r_1}} - \frac{\sum_{i\in N_{r_2}} z_i}{D_{r_2}} \leq \varepsilon_1 \qquad \forall r_1, r_2 \in R, \tag{5}$$

where

$$D_r = \left(\sum_{q\in Q_r} f_q \Big/ \sum_{q\in Q} f_q \right) \times 100$$

for each of $r = r_1$ and $r = r_2$. In this constraint, the demand of a region, D_r, is defined as the percent of the traffic volume departing from or arriving at the region. The number of refueling stations in each region is determined by the demand of each region, and given an equity parameter ε_1. The greater the traffic demand in a region, the more refueling stations in that region.

Equity constraint (5) compares the number of refueling stations in any two regions to achieve an equitable distribution of refueling stations. We require $|R|(|R|-1)/2$ constraints of (5) in the model. This may increase the computation time significantly for a large-sized network or in networks with small equity parameters. Keeping the same idea on the notion of demand equity across regions, we propose an alternative form as follows:

$$p\frac{D_{r_1}}{D} - \varepsilon_2 \leq \sum_{i\in N_{r_1}} z_i \leq p\frac{D_{r_1}}{D} + \varepsilon_2 \qquad \forall r_1 \in R, \tag{6}$$

where

$$D = \sum_{r\in R} D_r.$$

In constraints (6), the number of refueling stations in each region is determined by the demand in that region. The quantity $p\frac{D_{r_1}}{D}$ represents the number of charging stations to be installed in region r_1 in proportion to the ratio of the travel demand from and to region r_1 to the total demand over all regions. While (5) constraints the relative number of charging stations in a region compared to other regions, the alternative form (6) constraints the absolute number of charging stations in each region to be proportional to its travel demand.

Constraints (5) can be replaced by constraints (6), even though the two forms are not mathematically equivalent. By adjusting the value of ε_2, the meaning of equity (based on the demand ratio) in (6) can be similarly interpreted to the meaning of equity in (5). The advantage of the

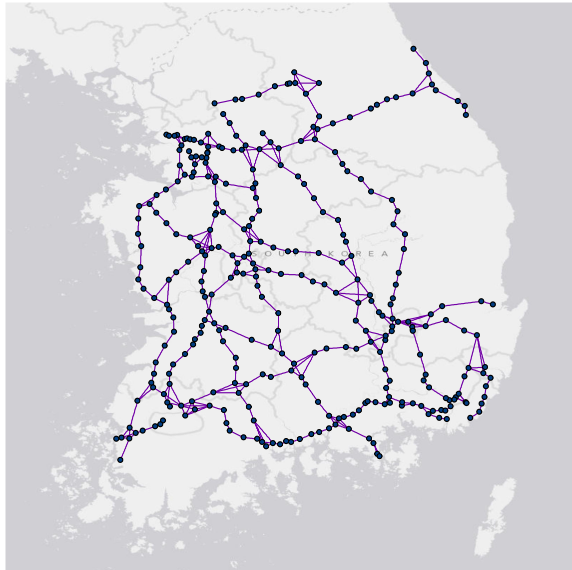


Figure 2: The Korean Expressway network [5]

reformulated model is that only $2|R|$ constraints are required to solve the problem, which can decrease the computation time.

3.3 Flow Equity Constraints

The second form of equity constraints ensures similar percentage of flows that pass through each region is captured by refueling stations.

$$\frac{\sum_{q\in\overline{Q}_{r_1}} f_q y_q}{\sum_{q\in\overline{Q}_{r_1}} f_q} - \frac{\sum_{q\in\overline{Q}_{r_2}} f_q y_q}{\sum_{q\in\overline{Q}_{r_2}} f_q} \leq \varepsilon_3 \qquad \forall r_1, r_2 \in R. \tag{7}$$

This constraint maintains the ratio of traffic volume that is refueled and able to pass through a region to the total traffic volume that wants to pass through the region. When the traffic flow passing through nodes in certain regions increases, the number of refueling stations in those regions should increase. Similar to the previous constraint, the degree of equity is determined by ε_3. The smaller ε_3 is, the more equitable the distribution of refueling stations is.

4 Solution Methods and Numerical Experiments

We use data from the Korean Expressway network to compare AC-PC FRLM and the proposed models. To use the Korean Expressway, a driver must pass through tollgates when entering and exiting the expressway. The toll amount that a driver pays at the destination tollgate corresponds to the distance between the driver's origin and destination. Using this data, traffic volumes for all OD pairs are gathered. The Korea Expressway Corporation manages all the data from the Korean Expressway. The Korean Expressway network consists of 324 nodes, 880 arcs, and 104,652

Table 2: Number of nodes in each region

Region (name)	Number of nodes
1 (Gyeonggi-do)	49
2 (Gangwon-do)	28
3 (Chungcheongbuk-do)	28
4 (Chungcheongnam-do)	43
5 (Gyeongsangbuk-do)	47
6 (Gyeongsangnam-do)	64
7 (Jeollabuk-do)	32
8 (Jeollanam-do)	33

OD pairs. The network is divided into eight regions, according to administrative areas. Figure 2 represents the Korean Expressway network, and Table 2 shows the number of nodes in each region. The number of OD pairs with a traffic flow of less than 15,000 is 98,392, comprising 94.02% of all the OD pairs. The sum of the traffic volume of each of these OD pairs, however, is 87,426,590, which is only 9.10% of the total traffic volume. We exclude these OD pairs from our analysis to reduce computation time (Chung and Kwon, 2015). The demand data used in our experiment is a network with 6,260 (5.98%) OD pairs, covering 90.90% of total traffic flow.

For computation and analysis, we used GAMS 24.7.3 with CPLEX 12.6 solver on a 2.20GHz Intel Core i5 CPU and 8.00 GB RAM computer. We tested various CPLEX options and used the following option values for the best performance:

- MIP starting algorithm (`startalg`): barrier
- Node selection strategy (`nodesel`): best-sestimate search
- Variable selection strategy at each node (`varsel`): branch based on pseudo reduced costs

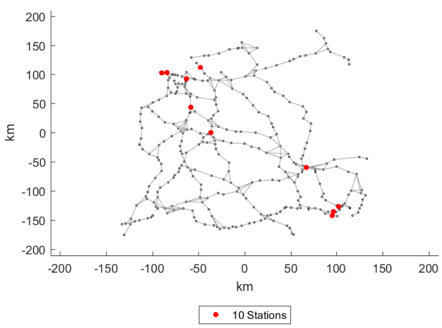
For the dataset, we use the set-covering AC-PC model of Capar et al. [3] to compute the minimal number of charging stations for 100% coverage. When the driving rage is 80 km, we need $p = 154$ stations, for 100 km $p = 153$, and for 120 km $p = 153$.

4.1 AC-PC FRLM Problem Without Equity Constraints

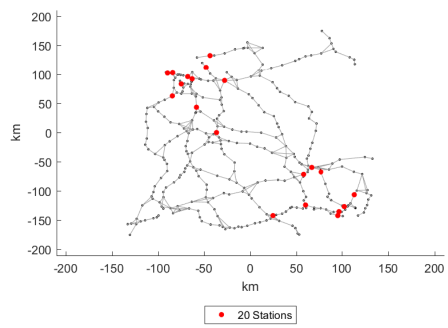
We first solve the original AC-PC FRML problem without equity constraints. Table 3 shows the results of AC-PC FRLM with various driving ranges (80 km, 100 km, and 120 km), and the number of refueling stations to be implemented (10, 20, and 30). As the driving range increases, EVs are able to travel longer distances without refueling, and some OD pairs may be traveled with fewer refueling stations. Both the percent of refueled traffic volume and the percent of refueled OD pairs increase as the driving range increases. When a driving range increases from 80 km to 120 km, with 30 refueling facilities, the percent of refueled traffic volume increases from 72.17% to 77.75%, and the percent of covered OD pairs increases from 46.95% to 57.01%. Computation time for the original problem is less than ten seconds.

Table 3: Results of AC-PC FRLM

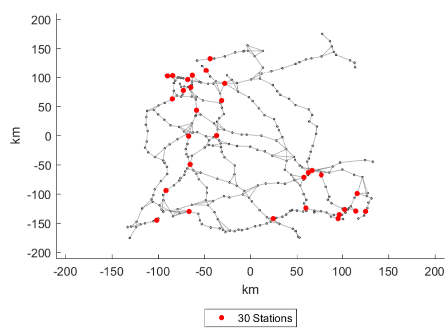
Driving range	p	Optimal solution	Percent of refueled traffic volume	Percent of refueled OD pairs	Computation time (seconds)
80 km	10	353,300,114	40.44%	18.59%	5.74
	20	520,211,313	59.54%	37.25%	5.92
	30	630,562,543	72.17%	46.95%	7.41
100 km	10	383,771,607	43.93%	19.11%	5.50
	20	552,118,040	63.19%	40.38%	6.27
	30	663,180,942	75.91%	51.13%	8.03
120 km	10	405,926,321	46.46%	25.05%	4.15
	20	579,114,054	66.28%	43.26%	6.30
	30	679,317,590	77.75%	57.01%	5.99



(a) $p = 10$

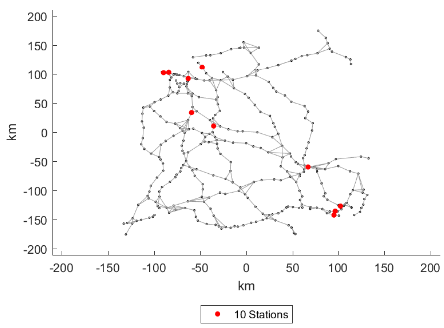


(b) $p = 20$

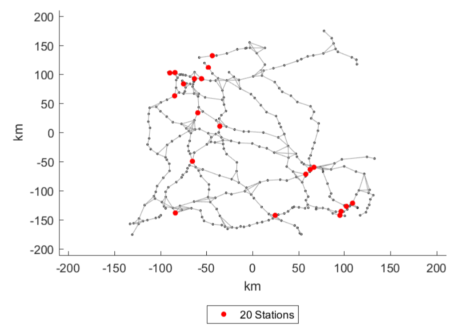


(c) $p = 30$

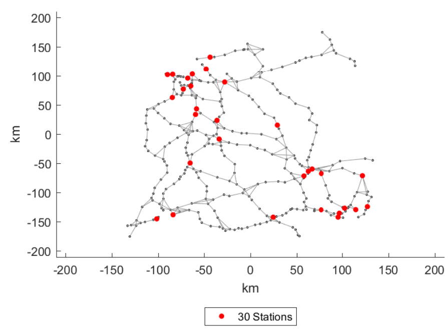
Figure 3: Results of AC-PC FRLM with a driving range of 80 km



(a) $p = 10$



(b) $p = 20$



(c) $p = 30$

Figure 4: Results of AC-PC FRLM with a driving range of 120 km

Figures 3 and 4 show optimal locations for AC-PC FRLM when the driving ranges are 80 km and 120 km, respectively. These figures show that most refueling stations are located in the northwestern (region 1) and southeastern regions (regions 5 and 6) of the Korean Expressway, which correspond to metropolitan areas in South Korea. When the driving range is 120 km and 30 refueling stations are implemented, the northwestern region has 10 refueling stations and the southeastern regions have 13 refueling stations. In other words, 23 refueling stations are located in three regions, which is 76.67% of the total number of refueling stations in the Korean Expressway network.

4.2 Demand Equity Constraints

In order to demonstrate how the proposed model performs, we solve AC-PC FRLM with demand equity constraints. It is assumed that the driving range is 120 km and the number of facilities is 30. Various equitable distribution networks of recharging stations are determined using the following steps:

Step 0: Determine initial parameter values of ε_1 and ε_2.

Step 1: Solve AC-PC FRML with demand equity constraints and calculate equity measure E_r of region r for all $r \in R$. If AC-PC FRML with demand equity constraints does not have a feasible solution, Stop

$$E_r = \frac{\sum_{i \in N_r} z_i}{D_r} \qquad \text{for equity constraint (5)}$$
$$E_r = \sum_{i \in N_r} z_i - p\frac{D_r}{\sum_{r' \in R} D_{r'}} \qquad \text{for equity constraint (6)}$$

Step 2: Calculate maximum deviation MD of equity measure among regions:

$$\text{MD} = \max_i E_i - \min_i E_i$$

Step 3: Update parameter values of ε_1 and ε_2, and go to Step 1.

$$\varepsilon_1 \text{ or } \varepsilon_2 = \text{MD} - \delta$$

where $\delta > 0$ is a small constant.

When the initial value assigned at Step 0 is large enough, the equity constraints are not binding and equity constrained AC-PC FRML model is equivalent to original AC-PC FRML model. Table 4 summarizes the objective values and computation times for AC-PC FRLMs with equity constraints (5) or (6). The first row shows that the optimal solution is the same as that of the original AC-PC FRLM when the equity parameters are sufficiently large. As the value of an equity parameter

Table 4: Results of AC-PC FRLM with demand equity constraints

with demand equity constraint (5)				with demand equity constraint (6)			
ε_1	Percent of refueled traffic volume	Percent of covered OD pairs	Comp time (seconds)	ε_2	Percent of refueled traffic volume	Percent of covered OD pairs	Comp time (seconds)
1.00	77.75%	57.01%	6.51	2.00	77.75%	57.01%	7.11
0.44	77.70%	57.49%	10.14	1.57	77.70%	57.49%	6.61
0.39	77.66%	58.48%	8.31	1.08	77.66%	58.48%	7.61
0.16	-	-	Inf.*	0.79	77.60%	56.98%	7.90
0.14	-	-	Inf.*	0.58	77.50%	57.09%	6.28
0.13	-	-	Inf.*	0.50	-	-	Inf.*

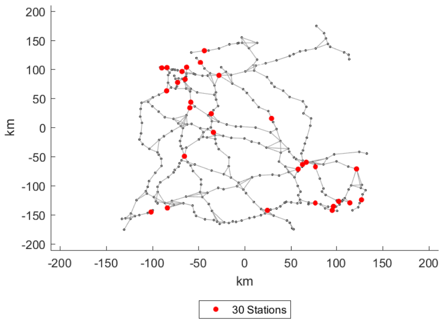
* A feasible solution does not exist.

decreases, the percent of refueled traffic volume decreases slightly. When ε_2 decreases from 2 to 0.58, the percent of refueled traffic volume decreases from 77.75% to 77.50%.

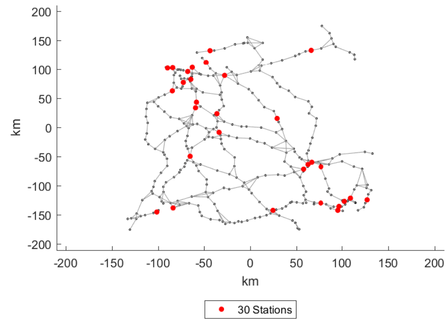
Interestingly, however, the percent of covered OD pairs does not decrease as equity parameter increases. To assure more equitable distribution of charging stations, the optimization procedure may give up to cover one OD pair with large travel demand in favor of many OD pairs with smaller travel demands. This finding suggests that people in more regions are served by the resulting refueling stations, even though the total captured traffic volumes decrease. Thus people in diverse locations have greater benefits and are able to visit more areas.

It is also observed that both equity constraints (5) and (6) require computation time around 7 to 10 seconds. With equity constraint (5), however, it is harder to find a feasible solution when the threshold value ε_1 is small.

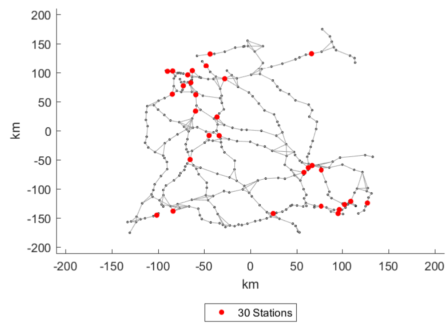
The equitable distribution networks of refueling stations from AC-PC FRLM with equity constraint (6) are shown in Figure 5. When $\varepsilon_2 = 2$, the result is the same as the result of the original AC-PC FRLM. The distribution of refueling stations becomes more equitable as ε_2 decreases. There is no refueling station in the northeastern region without considering equity. As ε_2 decreases from 2 to 0.58, refueling stations are consequently implemented in the northeastern region as well. The proportion of demand and refueling stations in each region are compared in Figure 6. The number of refuel stations in regions 5, 7, and 8 is relatively higher compared to the percentage of demand in the regions. Demand in region 5 is 13.6% of total demand, but the selected nodes for refueling stations are 20%. In contrast, the number of refuel stations in regions 1, 2 and 6 is relatively lower than others. With the demand equity constraint, the number of refuel stations becomes proportional to the demand.



(a) $\varepsilon_2 = 2$



(b) $\varepsilon_2 = 1.08$



(c) $\varepsilon_2 = 0.58$

Figure 5: Results of AC-PC FRLM with demand equity constraint (6)

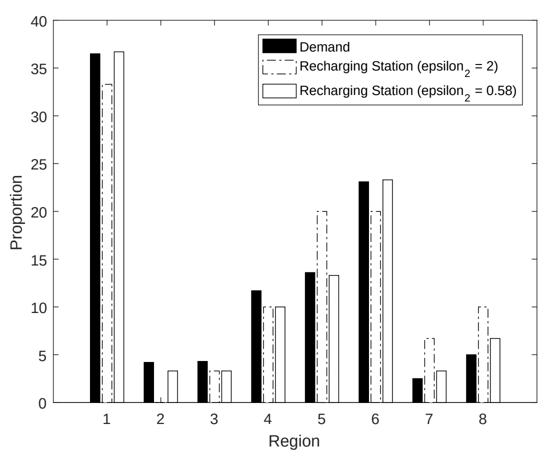


Figure 6: Number of refueling stations in each region

Table 5: Results of the n-phase method with $\varepsilon_3 = 0.1$

n	P	Objective value	Computation time (Sec.)
2	{25, 25}	776,560,908	162.19
3	{10, 15, 25}	776,560,908	158.44
4	{10, 10, 5, 25}	776,560,908	162.58
2	{30, 20}	776,351,168	14.45
3	{15, 15, 20}	776,351,168	18.91
3	{10, 20, 20}	776,351,168	17.62
4	{10, 10, 10, 20}	776,351,168	22.54

4.3 Flow Equity Constraint

When we utilize the AC-PC FRLM with flow equity constraint (7), computation time increases significantly as ε_3 decreases. Below a certain point of ε_3, the model cannot reach an optimal solution within a reasonable amount of time. To resolve this issue, we propose a simple n-phase method. The basic idea of the n-phase method is maximizing total transportation flow in the first $n - 1$ phases and then maintaining equity between regions in the final phase.

Step 0: Determine $n \geq 2$ and initial value of $P = \{p_1, p_2, ..., p_n\}$, where $\sum_{\tau=1}^{n} p_\tau = p$. Set $\tau = 1$.

Step 1: (Phase τ) Given p_τ, solve the original AC-PC FRLM problem to obtain the solution $\bar{z}^\tau$.

Step 2: Set $\tau = \tau + 1$. If $\tau < n$, fix $z_i = 1$ if $\bar{z}_i^{\tau-1} = 1$, and go to Step 1. If $\tau = n$, proceed to Step 3.

Step 3: (Final Phase) Given p_n, solve AC-PC FRLM with flow equity constraint.

This procedure will reduce the number of binary decision variables in each phase; hence it will decrease the computational time. Note that the last phase can be infeasible, if the flow equity constraint is too tight (ε_3 is too small) and/or there are insufficient number of charging stations to be located in the n-th phase (p_n is too small).

First, we compared the performance of the n-phase method by changing the number of phases, n. It is assumed that the number of recharging stations is $p = 50$ and ε_3 is 0.1. Table 5 shows that the number of charging stations in the last phase considering the flow equity constraint, p_n, slightly affect the performance of the solution. When $p_n = 25$ as presented in the first three rows in Table 5, the objective value is 776,560,908, while when $p_n = 20$, it is $776,351,168$. There is a significant difference in the computational time among the first three rows when $p_n = 25$ and the last four rows when $p_n = 20$. Since there are more combinations to consider in the feasible set as p_n increases, the computation time takes longer.

For further numerical experiments, we use $n = 2$, i.e., 2-phase method. We use $p_1 = 15$ and $p_2 = 15$, except the case of $\varepsilon_3 = 0.02$. When $p_1 = 15$ and $\varepsilon_3 = 0.02$, a feasible solution is not found in the second phase; we decrease the value of p_1 to 12, and find a feasible solution. Differently

Table 6: Results of AC-PC FRLM with flow equity constraint (7)

ε_3	Commercial Solver			2-Phase Method			
	Percent of refueled traffic volume	Percent of covered OD pairs	Computa-tion time (second)	Percent of refueled traffic volume	Percent of covered OD pairs	Computa-tion time (seconds)	Objective*
1	77.75%	57.01%	5.72	77.27%	56.10%	8.50	99.37%
0.5	77.66%	58.48%	30.21	77.22%	58.40%	10.30	99.43%
0.4	77.39%	55.83%	22.66	77.06%	55.94%	12.38	99.57%
0.3	77.16%	56.31%	43.62	76.75%	53.85%	33.37	99.46%
0.2	75.80%	59.78%	1183.70	75.50%	55.40%	172.35	99.61%
0.1	73.79%	53.79%	3306.29	73.72%	55.51%	303.07	99.92%
0.08	73.35%	57.81%	2218.44	73.24%	53.98%	211.04	99.86%
0.06	72.96%	55.85%	1364.90	72.55%	55.43%	1043.23	99.45%
0.04	72.42%	55.30%	3539.45	71.44%	55.46%	401.99	99.65%
0.02	71.63%	55.93%	≥36,000**	71.66%	55.96%	2537.48	100.02%

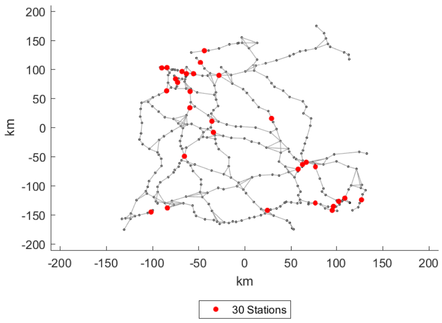
* The objective function value obtained by the 2-phase method, compared to the objective function value obtained by the commercial solver. 100.02% in the last row means that the 2-phase method found a better feasible solution than the commercial solver.

** An optimal solution was not found within 10 hours, but feasible solutions were found.

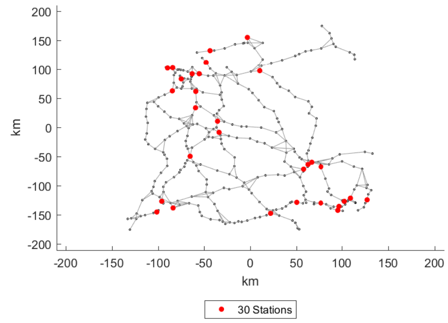
form demand equity constraints, more equitable solutions are obtained with various level of ε_3. A part of the results are summarized in Tables 6. When ε_3 is 0.02, solving the AC-PC FRLM with constraint (7) exceeds the computation time limit of 10 hours with a feasible solution found. However, we obtain a near optimal solution with optimality gap less than 1% in a shorter period of time with the 2-phase method ($n = 2$). The optimality gap in terms of objective function value is less than 1%, which means that the performance of the 2-phase method is very close to that of optimal solutions.

Figure 7 illustrates the location of refueling stations when ε_3 is 1, 0.3, and 0.1. Similar to the findings in the previous section, the refueling stations are distributed more equitably as ε_3 decreases. Figure 8 shows the percentage of captured traffic volume in each region. When $\varepsilon_3 = 1$, 89% vehicles in Region 1 are captured, but only 36% of vehicles in Regions 7. With the consideration of equity, however, the gap becomes closer. For example, when $\varepsilon_3 = 0.1$, 75% of vehicles in Region 1 and 67% of vehicles in Region 2 are served with the same number of recharging stations.

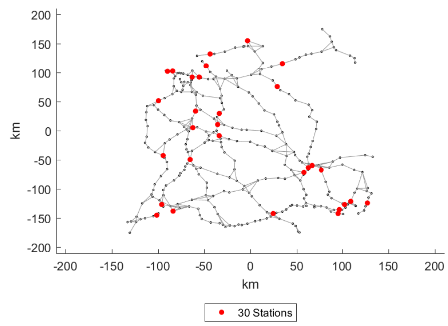
Figure 9 shows the tradeoff between efficiency in terms of the number of vehicles covered by the recharging stations and fairness in terms of allowed maximum flow equity measure, ε_3. There is no doubt that we need to sacrifice efficiency in order to improve equity in benefits from the recharging stations. However, it is worth to notice that efficiency is not a linear decreasing function of flow equity. The objective function value decreases slowly up to some point. That is, we can design equitable distribution networks without significant loss of efficiency.



(a) $\varepsilon_3 = 1.0$



(b) $\varepsilon_3 = 0.3$



(c) $\varepsilon_3 = 0.1$

Figure 7: Results of AC-PC FRLM with flow equity constraint

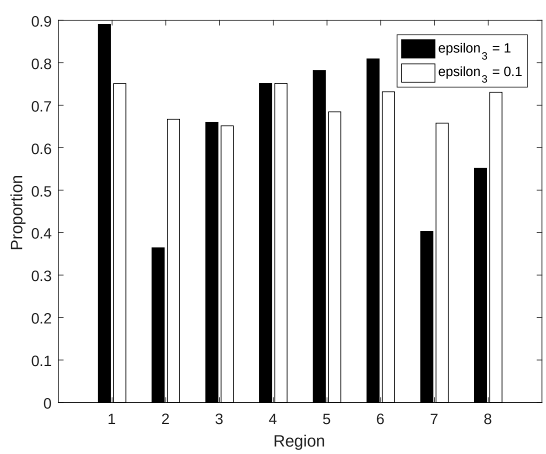


Figure 8: Flow equity among regions

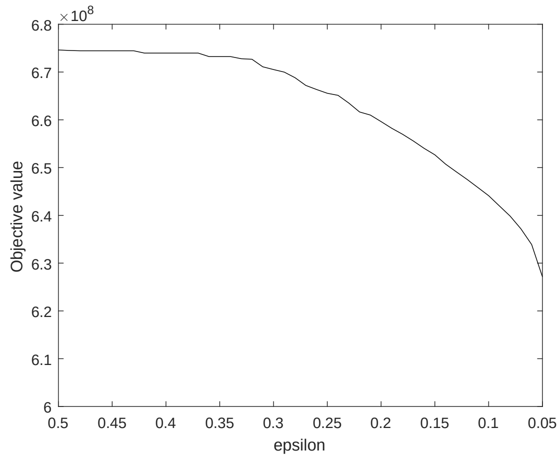


Figure 9: Tradeoff between efficiency and fairness

Table 7: Selected locations

	Equity tolerance levels	Percentage of refueled traffic volume	Percentage of covered OD pairs	Commonly selected nodes	Other nodes
AC-PC FRLM without equity constraint	N.A.	77.75%	57.01%	1, 9, 40, 76, 83, 86, 133, 165, 171, 175, 242, 271, 302, 317, 319	11, 91, 94, 95, 126, 160, 179, 186, 187, 200, 241, 248, 253, 275, 312
Demand equity constraint (6)	$\varepsilon_2 = 0.58$	77.50%	57.09%		91, 94, 95, 160, 179, 186, 187, 200, 209, 211, 238, 241, 248, 267, 312
Flow equity constraint (7)	$\varepsilon_3 = 0.1$	73.72%	55.51%		20, 73, 90, 99, 102, 160, 200, 201, 211, 226, 245, 248, 257, 280, 287
Both constraints (6) and (7)	$\varepsilon_2 = 0.59$, $\varepsilon_3 = 0.17$	69.12%	52.12%		20, 94, 102, 121, 192, 201, 211, 213, 243, 245, 247, 280, 301, 303, 323

4.4 Comparison of results

This section discusses the solution of the AC-PC FRLM models by comparing the location where recharging stations should be implemented to cover traffic flows. In addition to the previous models, we solve a AC-PC FRLM problem constrained by both demand equity constraint (6) and flow equity constraint (7). Equity tolerance levels ε_2 and ε_3 are set to 0.59 and 0.17, respectively. The results are summarized in Table 7. Interestingly, 15 nodes are commonly selected in all cases. When we compare the solution of AC-PC FRLM without equity constraints and the solution of demand equity constrained model, total 26 nodes are same. The flow equity constrained model generated 18 common nodes compared with the AC-PC FRLM without equity constraints. These results explain why the n-phase method finding some nodes without equity constraint and then applying equity constrained model for others is appropriate for the problem. Next, it is observed that considering both constraints in a model restrict the feasible region. When $\varepsilon_2 = 0.58$ and $\varepsilon_3 = 0.1$, the problem became infeasible. Relaxing the equity tolerance levels, an optimal solution was found but the percentage of refueled traffic volume and covered OD pairs were dropped significantly.

5 Conclusion

This paper develops three types of equity constraints for AC-PC FRLM and applies them to the Korean Expressway network: two based on demand and the other based on flow. The demand equity constraint compares the ratio of the number of refueling stations to demand in each region. In this case, the demand of a region is measured by the sum of traffic flows whose origin or destination nodes exist in the region. The flow equity constraint maintains the ratio of refueled traffic volume to the total traffic volume that needs to pass through the region.

When equity constraints are applied to large-scale networks and equity parameters decrease, however, the massive amount of computation time required to reach solutions is a problem. To address this issue, we reformulated the demand equity constraint and developed an n-phase heuristic method.

While the equity tolerance levels can be understood with their physical meanings, it is still difficult to determine a value. We expect that decision makers in practice will use various parameter values to generate diverse solutions, among which the decision maker will choose a solution that makes sense to him or her. Therefore, it is important to have an efficient computational method available on hand.

The results show that the original AC-PC FRML without equity constraints generates solutions in which refueling stations are located only in densely populated regions. With the proposed model, equitable distribution is achieved with a small drop of traffic volume refuels. Also, our model finds solutions within a reasonable amount of time.

There are several directions for further research. First, the present study does not consider the capacity of refueling stations. In reality, charging time is needed for refueling EVs, and the number of charging machines at any given refueling station can be limited. Second, this paper estimates travel demand based on current traffic flows. More accurate demand forecasting is essential for implementing refueling stations. Third, the proposed n-phase method can be further improved when combined with various local search methods, such as simulated annealing or tabu search. In our n-phase method, the decisions made in the prior phases are not modified in the current and future phases. Instead, we can consider small local changes to improve the solution quality. Finally, we can consider the availability of charging stations outside expressway networks. In this case, we can consider drivers' willingness to deviate from the shortest path and detour to recharge at those outside locations, and then we can also consider equity among OD pairs, instead of equity among regions.

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