Time-Dependent Hazardous-materials Network Design Problem

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Abstract

We extend the hazardous-materials (hazmat) network design problem to account for the time-dependent road closure as a policy tool in order to reduce hazmat transport risk by altering carriers' departure times and route choices. We formulate the time-dependent network design problem using an alternative-based model with each alternative representing a combined path and departure-time choice. We also present an extended model that can not only account for consecutive time-based road closure policies, but also allow stopping at the intermediate nodes of the network in the routing/scheduling decisions of the carriers. Heuristic algorithms based on column-generation and label-setting are presented. To illustrate the advantages that can be gained through the use of our methodology, we present results from numerical experiments based on a transportation network from Buffalo, NY. To investigate the impact of the extensions, we consider three versions of the problem by gradually refining the model. We show that under consideration of extensions, the design policies are more applicable and effective.

Keywords: hazardous materials transportation; network design; column generation; label setting

1 Introduction

Industrial development has tremendously increased the production of hazardous materials (hazmat) which itself necessitates large volumes of shipments. Even though hazmats are indispensable to our daily lives, their shipments can be disastrous to the people, environment and properties in the event of spillage, explosion and gas dispersion due to an accident. In an effort to protect our communities from hazmat incidents, studies on hazmat transportation focus on two main streams of research: risk measurement and hazmat routing. For the risk measurement literature, see Toumazis and Kwon (2015) and references therein.

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There are two types of hazmat routing studies: local routing and global routing (Erkut et al., 2007; Bianco et al., 2013). While local routing focuses on determining a safe route for a hazmat carrier, global routing deals with network regulation policies that encourage or enforce hazmat carriers to use safe routes. Hazmat-transport network design problems (HNDP) are for such global routing. This paper discusses HNDP in time-dependent road network settings.

For hazmat transportation, the problem of designing a network by imposing curfews using a bi-level programming formulation is first posed and studied by Kara and Verter (2004). They conclude that although government intervention in the route choices of carriers can achieve significant reductions in the transport risk, this might involve unbearable increases in carriers' transportation cost leading to unacceptable solutions for implementation. To account for the economic viability of the regulator's design policy from the viewpoint of carriers, Verter and Kara (2008) introduce a single-level path-based formulation for HNDP that helps generating mutually acceptable solutions.

Erkut and Alp (2007a) formulate HNDP as a minimum-risk tree design problem, and develop a greedy heuristic which allows the regulator to trade off risk and cost. Erkut and Gzara (2008) consider a similar problem to Kara and Verter (2004), and propose a heuristic method that improves the computational performance of the solution methodology. Considering link risk uncertainty, Sun et al. (2015) study the hazmat ND problem in a robust optimization framework. They show that, compared with the nominal solution, the robust solutions are superior in terms of risk reduction particularly in high-risk uncertainty and the worst case risk scenarios. Ignoring the role of the carriers, Bianco et al. (2009) solve a different bi-level HNDP with two types of authority actors where the goal is to determine the link capacities to minimize the risk in an equitable way.

Besides network design policies, an alternative hazmat regulation policy is toll setting which has been widely addressed in the literature as a policy for reducing the traffic congestion and regular freight transportation. In the field of hazmat transport regulation, TS was first proposed by Marcotte et al. (2009) as an effective tool to mitigate hazmat transport risk, and extended by Wang et al. (2012), Bianco et al. (2015), and Esfandeh et al. (2015).

This paper studies *time-dependent* HNDP (TD-HNDP), while most existing approaches assume that the underlying network parameters are static or time-invariant. According to the hazmat transportation statistics provided by U.S. Census Bureau (2012), 98.2% of explosives, 59.7% of flammables, 57.8% of oxidizers, and 26.1% of the toxic materials use truck as the mode of transportation with the average miles traveled per shipment being 576, 210, 245, and 302 miles, respectively. Thus many hazmat shipments are subject to travel over long distances, and consequently are likely to encounter a dynamic network with time-of-day variations in the parameters such as population and the regular traffic flow. Recognition of these time-varying patterns subsequently allows for time-varying estimations of hazmat risk exposure. Therefore, imposing a time-based road-closure may be sufficient to achieve significant reductions in transport risk.

Time-based curfew is a policy that has been both studied and applied in other areas of research such as aircraft scheduling for reducing noise disturbance and traffic congestion. One such example is the night flying restriction at London Heathrow airport. In the field of hazmat transportation, however, the time-based closure of certain roads to hazmat carriers has attracted the attention of the regulators only fairly recently. For instance, a time-based curfew on streets of Melbourne, Australia has restricted all trucks from using certain roads between 8 pm and 6 am on weekdays addressing concerns around noise, safety and air quality (Roads Corporation of Victoria (VicRoads), 2015). Nevertheless, we are not aware of the use of this policy tool in any hazmat regulation literature.

In the context of network capacity expansion, time-dependent network design problems have been studied (Szeto and Lo, 2008; Hosseininasab and Shetab-Boushehri, 2015; Miandoabchi et al., 2014; Lo and Szeto, 2009), wherein the scale of each time period is typically a year and the time-dependency results from the expanded road capacity and the new traffic equilibrium pattern. In this paper, however, the time-dependency comes from the time-dependent travel-time and hazmat risk within each day that is exogenous to the network design problem. Moreover, unlike capacity expansion problems, our decision can open and close road segments multiple times within each day. In that sense, the nature of our decision variable is closer to that of dynamic congestion pricing problems (Friesz et al., 2007; Do Chung et al., 2012), in which toll prices can vary multiple times within each day. We note, however, that road pricing is determined by a continuous decision variable, while curfew design is determined by a discrete decision.

In this paper, we investigate the *time-dependent road-closure* policy as a tool to temporarily prohibit the hazmat carriers from traversing certain road segments so as to mitigate the total population exposure. Our main contributions include:

1. We introduce *time-dependent hazmat network design problems* (TD-HNDP) which incorporate the risk implications of the carriers' least-cost route-time decisions into the design using a bi-level framework.

2. We employ a set of alternatives for each shipment in constructing our model, with each alternative being composed of a route and a departure time choice. This provides a single-level alternative-based formulation of our bi-level framework. We propose a column generation-based heuristic which generates the set of alternatives for each shipment by separately incorporating the perspectives of the regulator and the carriers.

3. To directly address the carriers' concerns around the economic viability of the design policies, we use a set of constraints that restrict the regulator's ability from maximally mitigating the transport risk.

4. To make time-dependent road-closure policies practically useful, we consider closures that are consecutive in time as well as stops en route that allow truck drivers to rest and meet the safety regulations.

The rest of this paper is organized as follows. Section 2 develops the time-dependent hazardous-network design model, encompassing the problem description, the mathematical formulation, and analyzing the obstacles towards solving the model. The development of a solution procedure driven from the model properties is outlined in Section 3. In Section 4, we present an extended model that

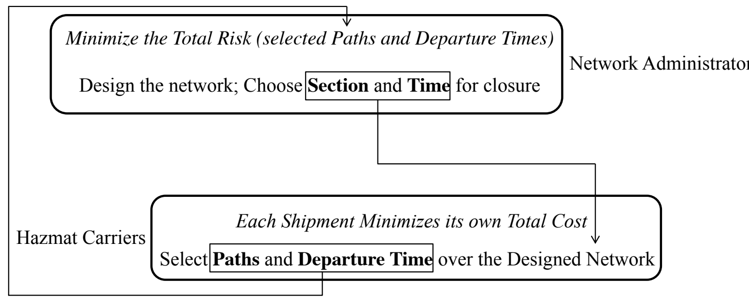


Figure 1: The Bi-level Structure of Network Design for Hazmat Transport

can handle consecutive road-closures and stops en carriers' route, and propose a computational method. Computational experiments are provided in Section 5. We present applications of our proposed methodology on the realistic road network of Buffalo, NY, and through this demonstrate the benefits of utilizing a time-dependent framework. We conclude the paper in Section 6 and suggest our future research directions.

2 The Time-Dependent Hazmat Network Design Problem

In our model, we let the regulator specify a subset of arcs on the region of interest which are subject to curfews. For instance, bridges, tunnels, and highly congested roads, which are more likely to be risky, might be allowed to be closed. These arcs can be categorized into a set of *sections* denoted by $\mathcal{S}$, according to a property specified by the regulator such that they are mutually exclusive but not necessarily collectively exhaustive. One such example is a section composed of a bidirectional road segment with both directions being in a similar geographical region, and consequently most probable to share common properties in terms of structure and population density. On the time-dependent network, the design policy then becomes to identify the sections that should be closed to hazmat trucks, temporarily at certain times. In fact, a design policy may include multiple *section-time closure* decisions.

There are two distinct groups of decision makers in the road network of interest: the network administrator, i.e., a local government, and the hazmat carriers. The regular traffic on the road network, however, are assumed not to be at the influence of the government agency and their flow is known a priori. We assume that the government has a leader position concerning with the hazmat transport risk imposed to the *population* and the *regular vehicles*, whereas the carriers attempt to minimize their travel cost while following the regulations. The policy available to the regulator is the authority to prevent the trucks carrying hazmat from using certain *sections* of the road network at certain *closure times*. On the other hand, hazmat carriers are allowed to select their *route* and *departure time* on the remaining hazardous-network. Such leader-follower relationship gives rise to a bi-level framework with the regulator's *section-time* decision in the upper level and the carriers' *route-time* choices in the lower level as depicted in Figure 1.

Once the regulator decides on section-time closure policies in the first level, the hazmat carriers will take the least-cost route-time alternatives between their origins and destinations in the second level. We note that despite the road network is being designated by the regulator, the actual risk is identified by the carriers' route-time choices over the remaining network which would not necessarily guarantee to be the regulator's desired least-risk solution. A bi-level framework allows the government to account for the risk implications of the users' route-time choices in designing the hazardous-network. We seek a *section-time* closure policy that is not only attractive for the regulator but also is economically viable to the carriers.

Since the nature of decision-makings in HND is inherently bi-level, many models are available in the form of bi-level optimization; for example Kara and Verter (2004), Bianco et al. (2009), Erkut and Alp (2007a), and Sun et al. (2015). These models assume that the hazmat carriers, who are the lower-level decision-makers, solves an optimization problem to find the shortest path given the network design policy from the upper-level. When the set of all alternatives available to the lower-level decision-makers can be enumerated, one can formulate an equivalent single-level optimization problem that models the bi-level decision-making process. In the static setting, Verter and Kara (2008) formulated a single-level optimization problem using the set of all enumerated paths. Note that in the bi-level optimization approach, we can avoid enumerating all available paths to model the bi-level decision-making process.

In this paper, we provide a single-level alternative-based formulation wherein the set of all possible route-time alternatives is determined a priori for each hazmat shipment, developing upon the work of Verter and Kara (2008). Such a single-level model is more effective than a link-based bi-level counterpart as it allows us to embed the time factor into the model leading to much simpler notation and model as well as handle practical factors in more complicated modeling.

2.1 Notation and Assumptions

Consider a time-varying transportation network during time interval $[t_0, t_f]$, where the existing road system is represented by the directed graph $\mathbb{G}(\mathcal{N}, \mathcal{A})$, with a set of nodes $\mathcal{N}$, and a set of arcs $\mathcal{A}$. We employ a discrete time index set as available traffic data for the majority of existing road networks is discrete. In particular, we let the entire time period, i.e. $[t_0, t_f]$, be divided into a number of sufficiently small time intervals of length Δ, each indexed by t. The details of this procedure are described in Appendix A.

We let $\mathcal{C}$ denote the set of all hazmat shipments across the network, each emerging at a certain time. Different hazmat types are distinguished by their corresponding radius of contamination when a hazmat incident occurs. The set of hazmat types (each exposing the network to a certain level of risk) is represented by $\mathcal{H}$. Consequently, each shipment $c \in \mathcal{C}$ is characterized by its origin $o(c) \in \mathcal{N}$, destination $d(c) \in \mathcal{N}$, the type of hazmat carried $h(c) \in \mathcal{H}$, as well as the earliest possible departure time from the origin $t_0^c \in [t_0, t_f]$. We let n^c represent the number of trucks consolidated in the same group to complete every shipment $c \in \mathcal{C}$. Table 1 provides the notation of the parameters and decision variables used in the formulations.

We make the following assumptions. We assume that the regular traffic flow in arc (i, j) during the

Notation	Description
$\mathcal{A}$	Set of arcs
$\mathcal{N}$	Set of nodes
$\mathcal{S}$	Set of sections subject to restrictions
$[t_0, t_f]$	Time period of interest
Θ	Discretized time period of interest, indexed by t
Δ	Length of the time intervals
$\mathcal{C}$	Set of hazmat O-D shipments
$\mathcal{H}$	Set of hazmat types
n^c	Number of trucks for shipment $c \in \mathcal{C}$
$o(c)$	Origin node of shipment c, $o(c) \in \mathcal{N}$
$d(c)$	Destination node of shipment c, $d(c) \in \mathcal{N}$
$h(c)$	Hazmat type transported by shipment c, $h(c) \in \mathcal{H}$
t_0^c	The earliest possible departure time for shipment c from origin $o(c)$, $t_0^c \in \Theta$
λ_h	Radius of contamination/evacuation when a type h hazmat accident occurs, [mile]
$\mathcal{P}^c$	Set of routes available for shipment c, from $o(c)$ to $d(c)$, indexed by p
$\mathcal{K}^c$	Set of route-departure time alternatives available for shipment c, from $o(c)$ to $d(c)$, indexed by k
r_k^c	Hazmat risk exposure when shipment c chooses alternative k
$\tau_{ij}(t)$	A nonnegative travel time for arc $(i,j) \in \mathcal{A}$, when departing from node i at time t
$f_{ij}(t)$	Average regular traffic flow in arc $(i,j) \in \mathcal{A}$, when departing from node i at time t
$r_{ij}^h(t)$	Risk exposure of a truck carrying hazmat h on link (i,j), departing from node i at t
ρ_{ij}	Population density on arc (i,j), [# of people per sq. mile]
ρ_{ij}^h	Number of people exposed on arc (i,j) when a truck is carrying hazmat type h
Pr_{ij}	Hazmat accident release probability along arc (i,j) per mile traveled
ω_{st}^h	Binary variable taking 1, if section s is closed to hazmat type h shipments at time t, 0 otherwise
y_k^c	Binary variable taking 1, if alternative k is available for shipment c, 0 otherwise
x_k^c	Binary variable taking 1, if alternative k is used for shipment c, 0 otherwise
δ_{sth}^{kc}	Equals 1, if alternative k of shipment c becomes unavailable when closing section s at time t to hazmat type h

Table 1: Mathematical notation

time interval t, $f_{ij}(t)$, is a known parameter and exogenous to the model. For analytical tractability of our model, two simplifying assumptions are also made; First, our model is deterministic. That is, there is no stochasticity either in the network parameters or the behavior of users. Second, we assume that hazmat carriers have perfect information of the time-dependent status of the network, including regular traffic and road bans. The final behavioral assumption is that the carriers never enter a road segment when they cannot exit it in time to respect a specified time-dependent road-closure. That is, an arc (i, j) can be entered at time t only if it is open during the entire $\tau_{ij}(t)$.

2.2 Definitions

We establish three fundamental definitions based upon which we construct our model: *the section-time closure decision*, *the route-time alternative*, and *the alternative set*. We present the formal description of each term in the following.

In our time-dependent model, the design policy is to identify sections that should be closed to hazmat trucks, temporarily at certain times, or to make*section-time closure* decisions, defined as follows:

Definition 1 (Section-Time Closure)**.** A *section-time closure* decision is comprised of a section $s \in \mathcal{S}$ and a time index $t \in \Theta$, and denotes closing all arc components of s during $[t, t + \Delta]$.

Note that it is possible to let each section $s \in \mathcal{S}$ correspond to a link $(i, j) \in \mathcal{A}$ with $|\mathcal{S}| = |\mathcal{A}|$. To build a single-level formulation, we need the following two definitions:

Definition 2 (Route-Time Alternative)**.** For a shipment c from the origin $o(c)$ to the destination $d(c)$, and the earliest departure time from the origin node as t_0^c with $t_0^c \in \Theta$, a *route-time alternative* k is characterized as the combination of a route $p \in \mathcal{P}^c$ and a departure time from the origin t such that $t \in \Theta$ and $t \geq t_0^c$, where $\mathcal{P}^c$ corresponds to the set of all routing options between $o(c)$ and $d(c)$.

Definition 3 (Alternative Set)**.** For a shipment c from the origin $o(c)$ to the destination $d(c)$, *the alternative set* $\mathcal{K}^c$ consists of all possible route-time alternatives as defined in Definition 2, excluding those with arrival time at $d(c)$ beyond t_f.

There are two attributes associated with each route-time alternative, namely the risk and cost of the alternative. Comprehensive descriptions on calculating these two attributes are delegated to Appendices B and C. The cost of an alternative includes penalties for deviating from the preferred departure time (PDT) and the preferred arrival time (PAT).

2.3 The Model

Similar to the single-level model of Verter and Kara (2008) that relies on the set of available paths, our proposed model relies on the set of route-time alternatives for each shipment c, i.e., $\mathcal{K}^c$. The alternatives in $\mathcal{K}^c$ are listed in increasing order of their corresponding cost incurred by the carriers. That is, alternative $k \in \mathcal{K}^c$ denotes the k-th preferred—k-th least-cost—alternative for carriers of shipment c. When there is a tie, we give priority to the alternative with higher risk; this will help us to obtain more robust solutions. The TD-HNDP is stated as follows:

$$\text{(TD-HNDP)}: \quad \min_{\omega_{st}^h} z(x_k^c, \omega_{st}^h) = \sum_{c \in \mathcal{C}} \sum_{k \in K^c} r_k^c x_k^c + \epsilon \sum_{s \in \mathcal{S}} \sum_{t \in \Theta} \sum_{h \in \mathcal{H}} \omega_{st}^h \tag{1}$$

subject to

$$\sum_{h \in \mathcal{H}} \sum_{t \in \Theta} \omega_{st}^h \leq \overline{N}, \qquad \forall s \in \mathcal{S} \tag{2}$$

$$\sum_{k \in \mathcal{K}^c} x_k^c = 1, \qquad \forall c \in \mathcal{C} \tag{3}$$

$$y_k^c \leq 1 - (\delta_{sth}^{kc} \omega_{st}^h), \qquad \forall c \in \mathcal{C}, \forall k \in \mathcal{K}^c, \quad \forall t \in \Theta, \forall s \in \mathcal{S}, h = h(c) \tag{4}$$

$$y_k^c \geq 1 - \sum_{s \in \mathcal{S}, t \in \Theta} \delta_{sth}^{kc} \omega_{st}^h, \quad \forall c \in \mathcal{C}, \forall k \in \mathcal{K}^c, \quad h = h(c) \tag{5}$$

$$x_k^c \leq y_k^c, \qquad \forall c \in \mathcal{C}, \forall k \in \mathcal{K}^c \tag{6}$$

$$x_k^c \geq y_k^c - \sum_{\ell=1}^{k-1} y_\ell^c, \qquad \forall c \in \mathcal{C}, \forall k \in \mathcal{K}^c \tag{7}$$

$$x_k^c, y_k^c \in \{0, 1\}, \qquad \forall c \in \mathcal{C}, \forall k \in \mathcal{K}^c \tag{8}$$

$$\omega_{st}^h \in \{0, 1\}, \qquad \forall s \in \mathcal{S}, \forall t \in \Theta, \forall h \in \mathcal{H}. \tag{9}$$

The second term in the objective function is to avoid closing unnecessary sections; ϵ is a sufficiently small positive constant. We note that TD-HNDP is a linear optimization problem with binary variables, which can be solved by an optimization solver like CPLEX.

Three sets of decision variables are employed in TD-HNDP. Variables ω_{st}^h represent the section-time closure decisions of the regulator. The outcome of the regulator's decisions in terms of the available alternatives in $\mathcal{K}^c$ are captured by the y_k^c variables. Ultimately, the carriers' route-time decisions on the remaining network are represented by the x_k^c variables. Overall, TD-HNDP adopts the viewpoint of the regulator. The administrator identifies a section-time closure policy that minimizes the total hazmat transportation risk, taking into account that the carriers follow least-cost route-time decisions on the remaining network. Constraints (3)–(8) capture the the carriers' lower-level decision-making depicted in Figure 1.

Constraints (2) state that a section cannot be closed for more than $\overline{N}$ time intervals, irrespective of the hazmat type. More frequent closures of a section refers to more government interventions in the hazmat network, and larger values of $\overline{N}$ are more likely to lead to unacceptable solutions to the carriers. Constraints (3) ensure that a single alternative is selected by each shipment. The impact of regulator's section-time closure decisions on the availability of the alternatives for each shipment is determined through constraints (4)–(5). In particular, constraints (4) express that an alternative with any of its links being closed at their corresponding entrance times due to at least one section-time closure cannot be selected by hazmat carriers. When none of the section-time closures can impact the alternative, i.e., all of its links are open at their corresponding entrance times, then constraints (5) assure availability of the alternative for hazmat shipments. Constraints (6) guarantee that only the alternatives made available by the government can be used by hazmat carriers. Given the ordering of $\mathcal{K}^c$, constraints (7) ascertain that, among the available alternatives for shipment c, the one with the smallest index would be used by the carrier. Finally, constraints (8)–(9) specify that the decision variables are binary.

One can notice that TD-HNDP enables the regulator to design a different network specific to each hazmat type, i.e., *hazmat-specific* network. However, it is possible to design a common network for all hazmat types, i.e., *single network*, by simply replacing ω_{st}^h with ω_{st}. Taking the argument to the other extreme, the model can be readily tailored to designate a *shipment-specific* network, where ω_{st}^h are replaced by ω_{st}^c.

A solution of the model not only prescribes the sections and their corresponding times to be closed to hazmat shipments by the regulator, but also determines the routes and departure times that would be traversed by each shipment on the designated network.

If the existing road network is connected, such that it enables the delivery of all shipments to their destinations, there is always a feasible solution to TD-HNDP. One extreme scenario is when $|\mathcal{K}^c| = 1$ for all shipments. In this case, the regulator is unable to intervene, leading to the carriers' most preferred (least-cost) alternative remain available for routing. This corresponds to the carriers' most desirable scenario. The other extreme, however, constitutes the optimal solution of TD-HNDP with the regulator's risk mitigation ability being maximized within the boundaries of the model parameters. This is captured when *all alternatives* are included in $\mathcal{K}^c$ for every $c \in \mathcal{C}$ [1]. Consequently, in order to guarantee the optimality of the solution to TD-HNDP, one needs to ensure that, for every shipment c, set $\mathcal{K}^c$ is comprehensive and is ordered appropriately. However, a significant issue pertaining to the development of this alternative-based model is the cardinality of the alternative set, i.e., $|\mathcal{K}^c|$.

Our TD-HNDP is at least as hard as the HNDP in static network. Amaldi et al. (2011) has shown that the link-based bi-level optimization formulation in static network is NP-hard. In the path-based equivalent reformulation of Verter and Kara (2008) and our alternative-based reformulation, the key challenge is to generate the sets of paths and alternatives. In the worst-case, we would need to enumerate all possible paths, which is known to be #P-complete (Roberts and Kroese, 2007); at least as difficult as NP-complete. This completes our analysis in motivating our solution methodology. In Section 3, taking advantage of the model analytical properties, we present a heuristic algorithm to solve TD-HNDP, wherein we generate a few alternatives in each iteration, instead of generating large number of alternatives in the beginning.

[1] Note that, unlike the path-based model of Verter and Kara (2008), maximizing the regulator's ability in the TD-HNDP framework is not identical to completely ignoring carriers' economic viability of the solutions. Recall that our model already restricts the level of government intervention by allowing for curfew-free links as well as by incorporating constraints (13).

3 Column-Generation-Based Heuristic

Exploiting specific analytical and structural properties of the TD-HNDP, we propose a heuristic algorithm that benefits from the notions of conventional column generation scheme, and can efficiently account for different perspectives of the regulator and the carriers.

Undoubtedly, one possible solution method for the TD-HNDP is to generate the comprehensive set of route-time alternatives for each shipment c by applying an enumerative procedure. Other than simplicity, explicit enumeration schemes guarantee optimality. However, for applications of realistic size, they are computationally prohibitive as the number of routes grows exponentially with the network size. Motivated by Verter and Kara (2008), another option is to include only the first k preferred alternatives of each shipment in the alternative sets, without ensuring the optimality of the solution. That is, for every shipment, one needs to calculate the k time-dependent least-cost routes between the shipment origin-destination pair. Although both the problems of finding k shortest paths between an origin-destination pair in static networks and time-dependent shortest paths in dynamic networks have been well studied in the literature, we are unaware of the existence of any efficient algorithm dealing with the k time-dependent shortest paths.

None of the above methods can be practically applied to TD-HNDP involving a large number of route-time alternatives. Consequently, a feasible method in this context is the incorporation of the column generation approach, which is a popular method for solving large-scale linear optimization problems pioneered by Dantzig and Wolfe (1960) and Gilmore and Gomory (1961). The details of this approach can be found in the comprehensive review by Lübbecke and Desrosiers (2005).

In our case, column generation approach decomposes the problem into two parts: a small-scale TD-HNDP with reduced number of route-time alternatives, and $|\mathcal{C}|$ subproblems each corresponding to a shipment. The reduced TD-HNDP identifies a design policy from already known feasible alternatives. With each column representing an alternative for a shipment, every subproblem is used to propose new feasible columns that improve the current solution of the reduced TD-HNDP. The main body of our column-generation-based heuristic has two standard building blocks: (i) the formulation of the restricted master problem, and (ii) the development of pricing subproblem structure. Apart from these, we also embed (iii) an auxiliary subproblem to reinforce the performance of the heuristic. In what follows, we present a formal statement of these three blocks along with their corresponding solution methods.

3.1 The Restricted Master Problem (RMP)

The master problem in our scheme is almost identical to TD-HNDP, where each decision variable x_k^c corresponding to a feasible route-time alternative represents a column for shipment c. Nonetheless, it differs from the TD-HNDP in that it is defined using only a subset of the feasible route-time alternatives for every shipment c, denoted by $\overline{\mathcal{K}}^c \subset \mathcal{K}^c$, thereby referred to as the restricted master problem (RMP). The linear programming relaxation of this reduced TD-HNDP is solved using linear programming solvers, e.g., CPLEX. We iteratively add new columns to $\overline{\mathcal{K}}^c$, potentially able to improve the objective of the RMP, by solving subproblems following each optimization of the RMP. We let the dual variables corresponding to constraints (3), (6) and (7) be denoted by γ^c, α_k^c, and β_k^c, respectively, which are transmitted to the pricing subproblem seeking to generate alternatives with favorable reduced costs.

We start our procedure by generating initial sets of route-time alternatives, one for each shipment. Let this initial subset of the feasible alternatives be denoted by $\mathcal{K}_1^c$ for shipment c. That is, $\overline{\mathcal{K}}^c$ is initially set to K_1^c. Such feasible alternatives always exist as the current road network is assumed to be connected assuring the delivery of all shipments to their destinations during the interval $[t_0, t_f]$. Although several local heuristics can be used to generate the initial alternatives, a simple yet never-fail method is to identify carriers' alternatives under having *no regulation*, i.e., the carriers ideal scenario. That is, with all sections open to shipments during the entire period of time, one needs to obtain the route-time alternatives inducing the minimum cost to the carriers. In particular, this task boils down to solving sets of time-dependent shortest-path (TDSP) problems, one for each shipment, with the cost being as (30) in Appendix C.

We employ the TDSP algorithm of Ziliaskopoulos and Mahmassani (1993) for generating an initial set of alternatives. Working on a discretized network, and based upon the Bellman's principle of optimality, Ziliaskopoulos and Mahmassani (1993)' algorithm calculates the time-dependent shortest path from every node in the network to a given destination node for every time step.

In our case, recall that the travel cost associated with an alternative, see equation (30), is in general dependent on the arrival time to the destination. This indeed prevents us from decomposing the cost of an alternative into its link-based components due to in every time step of the TDSP algorithm the total travel time of the route must be known a priori in order to calculate the deviation from the PAT. Under a parametric assumption specified in Appendix C, however, we notice that with the departure being fixed at time step t, the route with the shortest travel time also induces the minimum cost to the carriers; thereby the TDSP algorithm is readily implementable. That is, for every shipment c, it is sufficient to employ the TDSP algorithm, where travel time parameters constitute the arc impedances and the network is unregulated. In effect, this calculates, at every $\{t : t \in \Theta, t \geq t_0^c\}$, the time-dependent shortest path from $o(c)$ to $d(c)$ each denoting an alternative. The travel cost associated with each alternative is then identified by equation (30). Finally, the overall minimum-cost alternative is achieved by simply comparing the routes prescribed for all time steps with respect to their travel cost. This ends up with having at least one alternative in $\mathcal{K}_1^c$, for every shipment c, comprised of a route choice and a departure time choice. Once we generate these alternatives, the master and subproblems are solved iteratively. We now introduce the subproblems, which are solved repeatedly to find the potential candidate alternatives, to insert into the RMP.

3.2 The Sub Problems

In addition to the pricing subproblem which is an integral element of every column generation algorithm, we suggest embedding another set of subproblems, one for each shipment, referred to as the auxiliary subproblem. To improve the performance of the column generation in terms of computation effort and solution quality, the auxiliary subproblems are solved to generate more columns, following each optimization of the RMP. We now provide the descriptions of these two sets of subproblems.

3.2.1 The Pricing Sub Problem (PSP)

The Pricing Sub Problem (PSP) uses the duality concept in linear programming to find better candidate alternatives, comprised of a route and a departure time, to include in the RMP. The objective function of the PSP is the reduced cost of the new variable (i.e., alternative) according to the current dual variables. We let the RMP currently contain $|\overline{\mathcal{K}}^c|$ alternatives for shipment c, $|\overline{\mathcal{K}}^c| \geq 1$. Using these alternatives, suppose that we solve the RMP as an LP, and obtain optimal dual variables γ^c, α_k^c, and β_k^c associated with constraints (3), (6), and (7), respectively. Note that $\alpha_k^c, \beta_k^c \leq 0$, and γ^c is unrestricted in sign. The PSP seeks to determine an alternative with *negative reduced cost*.

We consider the existing road network $\mathbb{G}(\mathcal{N}, \mathcal{A})$, with the horizon of interest, $[t_0, t_f]$, being discretized. We also let the current time-varying risk parameters constitute the time-dependent arc impedances. Each time after solving the RMP, however, one needs to revise the arc impedances of the PSP based on the current dual variables. We now explain our approach to modify the arc impedances, and describe the algorithm used to solve the PSP.

Recall that the risk associated with a truck of shipment c when traversing arc (i, j) at time t, i.e., $r_{ij}^{h(c)}(t)$, is computed using equation (25) (see Appendix, Part B). To calculate the *modified risk* of using arc (i, j) at time t, we need an additional notation:

$$\varphi_{ijt}^{kc} = \begin{cases} 1 & \text{, if shipment } c \text{ enters arc } (i,j) \text{ at time } t \text{ when using alternative } k, \\ 0 & \text{, otherwise.} \end{cases}$$

Therefore, the modified arc risk, $\hat{r}_{ij}^{h(c)}(t)$, corresponding to a truck of shipment c when entering arc (i, j) at time t is given as:

$$\begin{aligned} \hat{r}_{ij}^{h(c)}(t) &= r_{ij}^{h(c)}(t) - \left(\sum_{k \in \overline{\mathcal{K}}^c} (+1) \varphi_{ijt}^{kc} \alpha_k^c + \sum_{k \in \overline{\mathcal{K}}^c} (-1) \varphi_{ijt}^{kc} \beta_k^c \right) \\ &= r_{ij}^{h(c)}(t) - \sum_{k \in \overline{\mathcal{K}}^c} \varphi_{ijt}^{kc} \alpha_k^c + \sum_{k \in \overline{\mathcal{K}}^c} \varphi_{ijt}^{kc} \beta_k^c \end{aligned} \tag{10}$$

Consequently, for shipment c, the reduced cost associated with alternative k, $\hat{r}_k^c$, can be obtained using an additive procedure analogous to the one in equation (26), with the link-based components replaced by equation (10). Note that the calculation is complete after subtracting the dual variables γ^c corresponding to shipment c. Therefore, for k composed of route p and departure time from the origin as t, the reduced cost is given as:

$$\hat{r}_k^c = -\gamma^c + \sum_{(i,j)\in\mathcal{A}^p} n^c \hat{r}_{ij}^{h(c)}(\phi_{ij}^p(t)) \tag{11}$$

with $\mathcal{A}^p \subset \mathcal{A}$ being the set of road links constituting path p, and $\phi_{ij}^p(t)$ calculated by equations (27)–(29). It is important to notice that while the original arc impedance, $r_{ij}^{h(c)}(\cdot)$, is a nonnegative number, the modified arc cost, $\hat{r}_{ij}^{h(c)}(\cdot)$, can take any real value.

All the alternatives transferred from this stage should satisfy (i) the precedence relations, and (ii) the time window. Further, as *loopless* routes are desirable in the context of hazmat shipments, the alternatives must be *elementary*. That is, all nodes along a route can be visited only once. Consequently, the subproblem for every shipment, i.e., PSP(c), is indeed a time-dependent elementary shortest path problem with resource constraints (TD-ESPPRC) with time-varying arc costs as given by equation (10).

We extend the idea of Nemani et al. (2010), who proposed a labeling algorithm for solving static ESPPRC, and propose a heuristic approach based on dynamic programming that solves the TD-ESPPRC by repeating a label setting algorithm for every departure time. Despite the time-dependent arc costs $\hat{r}_{ij}^{h(c)}(t)$ are unrestricted in sign, our algorithm enables us to find a set of good alternatives that have negative reduced costs and are elementary, even if the graph contains negative cost cycles.

The pseudo-code of our label setting algorithm used to solve each ESPPRC is depicted in Algorithm 1. Briefly, we associate with each partial path ending at node u, a label indicating the consumption of the resource (travel time) T_u, i.e., $\{u, T_u\}$. The cost of a label, $\mathsf{Cost}(\{u, T_u\})$, is calculated by adding the time-dependent modified risk of the arcs on the partial path. The predecessor nodes of each partial path are stored in the Preds array in order to prevent visit to nodes more than once in any extension of the corresponding label. Although there may be exponentially many labels, an efficient implementation of our algorithm is guaranteed by two observations. First, the feasibility check limits the number of the labels by detecting the nodes which cannot be visited in any extension of a label due to cycling or travel time limitations. Second, the dominance check eliminates the dominated labels by incorporating the basic dominance rule. Note that, for time saving purposes, a modified dominance rule which guarantees optimality of the algorithm as applied by Chabrier (2006) and Nemani et al. (2010) can be used only when the algorithm is not able to discover a column with negative reduced cost.

Finally, it is worth mentioning that every iteration of the column-generation-based heuristic involves solving $|\mathcal{C}|$ PSPs, each corresponding to a shipment. For every shipment, the modified arc risks are updated each time the RMP is solved. The current section-time closure policy obtained from the RMP, however, is not enforced to the network when solving these subproblems. That is, the new columns are generated from an unregulated network.

Algorithm 1 Label setting framework for solving ESPPRC at departure time t_0 for PSP (c)

Inputs: Updated time-dependent arc risks, $\hat{r}_{ij}(t)$, time-dependent arc times, $\tau_{ij}(t)$, precedence matrix, $o(c)$, $d(c)$, and departure time from the origin, t_0.
Output: A set of elementary routes for departure time t_0 for shipment c from $o(c)$ to $d(c)$ that do not violate the time limit constraint and have negative reduced costs.
Step 1: Initialization
    Get the updated arc risks associated with the last RMP solved,
    Create label $\{o(c), 0\}$,
    Add label $\{o(c), 0\}$ to the set of unprocessed labels, Φ, and set $\mathsf{Cost}(\{o(c), 0\}) = 0$,
    Initialize a set to store each route found,
    Initialize the predecessor set to store the predecessor nodes of each label,
    Set `isFeasible` $=$ `false`, and `isImproved` $=$ `false`.
Step 2:
while $u \neq d(c)$ **do**
  Find the Best Label:
  $\{u, T_u\} := \{\{u, T_u\} : T_u = \min\{T_w\}, \forall \{w, T_w\} \in \Phi, w \neq d(c)\}$
  if $t_0 + T_u \leq t_f$ **then**
    Find the set of instant neighbors of u, i.e., Γ_u
    for each $v \in \Gamma_u$ **do**
      Feasibility check:
      if $T_u + \tau_{uv}(t_0 + T_u) \leq t_f$ & $v \notin \mathsf{Preds}[\{u, T_u\}]$ **then**
        `isFeasible` $=$ `true`
        $T_v = T_u + \tau_{uv}(t_0 + T_u)$
        Existence check:
        if $\{v, T_v\} \notin \Phi$ **then**
          $\mathsf{Cost}(\{v, T_v\}) = \infty$
        end if
        Improvement check:
        if $\mathsf{Cost}(\{u, T_u\}) + \hat{r}_{uv}(t_0 + T_u) < \mathsf{Cost}(\{v, T_v\})$ **then**
          Dominance check:
          if $\nexists \{w, T_w\} \in \Phi$ such that $w = v$ & $T_w \leq T_v$ & $\mathsf{Cost}(\{w, T_w\}) < \mathsf{Cost}(\{v, T_v\})$ **then**
            Create label $(\{v, T_v\})$, and add it to the set of unprocessed labels, Φ
            $\mathsf{Cost}(\{v, T_v\}) = \mathsf{Cost}(\{u, T_u\}) + \hat{r}_{uv}(t_0 + T_u)$
            $\mathsf{Preds}[\{v, T_v\}] = \mathsf{Preds}[\{u, T_u\}] \cup \{u\}$
            `isImproved` $=$ `true`
          end if
        end if
      end if
      if `isFeasible` & `isImproved` & $v = d(c)$ & $\mathsf{Cost}(\{v, T_v\}) < 0$ **then**
        Store the distinct route from $o(c)$ to v in the route set.
      end if
    end for
    Remove label $\{u, T_u\}$ from the set of unprocessed labels, Φ
  end if
  Remove label $\{u, T_u\}$ from the set of unprocessed labels, Φ
end while

3.2.2 The Auxiliary Sub Problem (ASP)

The main idea of the Auxiliary Sub Problem (ASP) is to determine, for every shipment, the carriers' minimum-cost alternatives subject to the current section-time closure policy. That is, each time the RMP is solved, the design policy is imposed to the network, and the minimum-cost alternatives of shipment c are identified by solving ASP(c) with the procedure analogous to the one explained in Section 3.1 for the generation of initial alternatives.

It is clear that ASP is not only easy to solve, but also has the advantage of providing more columns to include in the RMP at each iteration which is beneficial for a fast evolving algorithm. More importantly, ASP captures the carriers' behavior when the design decisions are implemented which is the goal of the TD-HNDP. In fact, if the heuristic were made based solely on adding candidate alternatives from the PSPs, significant reductions in transport risk can be obtained through the design decisions prescribed by the model. On such a network, however, it is possible that less expensive alternatives than the minimum-risk ones would be available for some shipments. Thus, the regulator's ability to reduce the risk to the level prescribed by the heuristic is not guaranteed in practice. This also implies that the design policy obtained from the heuristic is less likely consistent with the exact solution of the TD-HNDP. Instead, ASP allows to rightly incorporate the carriers' responses to the design policy each time such a decision is made, thereby helping the heuristic to more likely provide a correct estimation of the resulting transport risk upon implementation.

We also notice that as the PSPs disregard the design decisions when seeking for new alternatives, it is possible that existing columns would be generated repetitively, leading to either the premature stop or infinite cycling of the algorithm. Employing ASP, however, ensures that at least a new column is added to the RMP at each iteration as long as the current columns do not include the carriers' real choice under the design policy, prohibiting the aforementioned issues.

3.3 The Structure of the Algorithm

The structure of the proposed column-generation-based heuristic implemented for TD-HNDP is depicted in Figure 2.

To obtain the final solution, one needs to solve the TD-HNDP with all columns generated while solving the RMP. This is achieved upon accomplishing the tasks in the dotted area of Figure 2. Nevertheless, for the rare cases wherein the integer solution of the TD-HNDP fails to detect the carriers' alternatives upon implementation, an ASP loop can be added to the procedure to ensure that the real performance of the provided design policy is indeed the one anticipated by the model.

In closing Section 3, it is worth emphasizing that even though, in general, column generation is considered as an exact algorithm for the LP relaxation of MILPs, a solution to the MILP produced by the generated set of columns is not necessarily optimal. This indicates that the column-generation-based algorithm is a heuristic approach for the TD-HNDP, not an exact one, which provides an upper bound to the problem. Further, in our case, we cannot either claim that the solution obtained for the relaxed master problem is optimal due to the following reason.

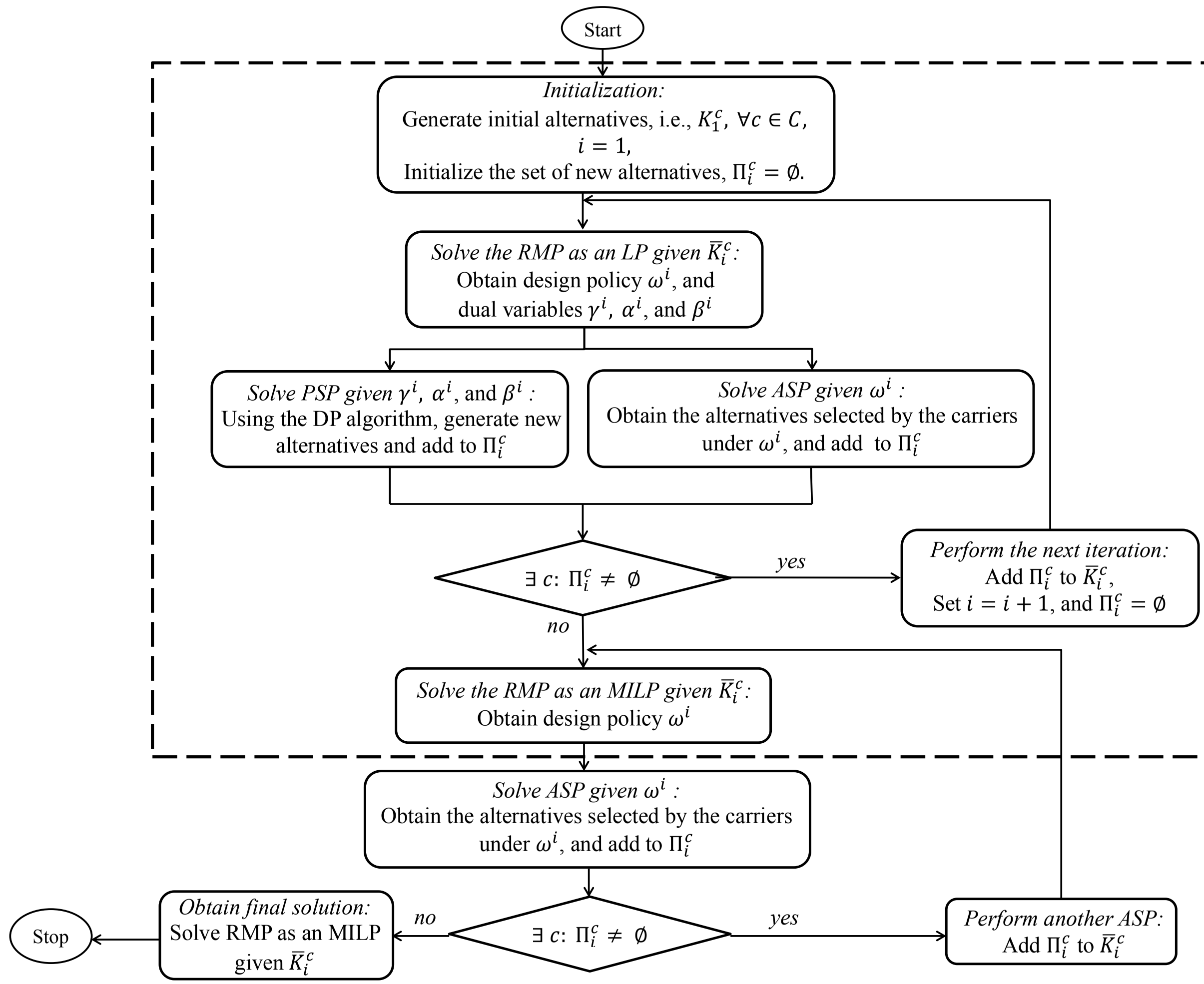


Figure 2: Flowchart of the proposed Column-Generation-Based procedure

The classical column generation framework assumes that the number of constraints in the restricted master problem is fixed, i.e., all constraints are known explicitly, and thereby complete dual information is passed to the PSP. However, one can observe from equations (1)–(9) that in our case the RMP has *column-dependent-rows*. In fact, four new constraints, namely constraints (4), (5), (6), and (7) are added to the RMP subsequent with generation of every new column. Therefore, during column generation, the RMP grows both vertically and horizontally. Since no dual information corresponding to the missing constraints (6) and (7) is supplied to the PSP, we are not able to claim that the reduced cost of a new column is accurately computed in the subproblem.

4 The Extended Model

The TD-HNDP identifies the time-based road closure policies, without a guarantee on the prescribed road closure policies to be sequential in time. The benefit of developing time-dependent road closure policies is described in Appendix D, in comparison to the static policies. Despite being effective for risk mitigation, non-consecutive closure of a road segment is not only difficult to be realized by the carriers, but also involves excess management cost devoted to each time step from a regulator's perspective. This limitation motivates us to refine the TD-HNDP to identify time-dependent road-closure policies which are consecutive in time.

Further, we have assumed that the carriers' driving times are within the regulator's mandate, negating the need for stopping along the way. In practical situations, however, many hazmat shipments are subject to travel over long distances and require more than one day to be delivered. Therefore, one or multiple intermediate stops may be necessary along the route due to refueling and drivers' workload constraints. Waiting at intermediate nodes not only expands the applicability of our hazmat network design model, it is also likely to be beneficial from a risk-minimization and cost-minimization perspective. With this motivation, we also analyze the TD-HNDP in which stop and waiting at a restricted set of nodes, i.e., rest areas, is allowed to hazmat trucks.

4.1 Consecutive Closures

The refinement of TD-HNDP in Section 2 to account for consecutive closures requires a modification to an existing assumption as well as introducing some additional definitions and notations.

The original model considers regulating a time-varying hazmat transportation network wherein every shipment can be completed within one cycle, i.e., the time interval of $[t_0, t_f]$. In the current work, however, the time-varying network is studied over a set of successive equal-length cycles, e.g., weekdays, denoted by $\mathcal{L}$, with each cycle $l \in \mathcal{L}$ representing a time interval of length $[t_0, t_f]$. This assumption is necessary to tackle regulating the hazmat shipments with long trip durations. Further, we let the time-varying patterns of the network parameters such as regular traffic flow be identical over all cycles. That is, the network conditions of every cycle are repeated in the subsequent cycle. This assumption requires the following modification to the notations of the existing parameters and decision variables.

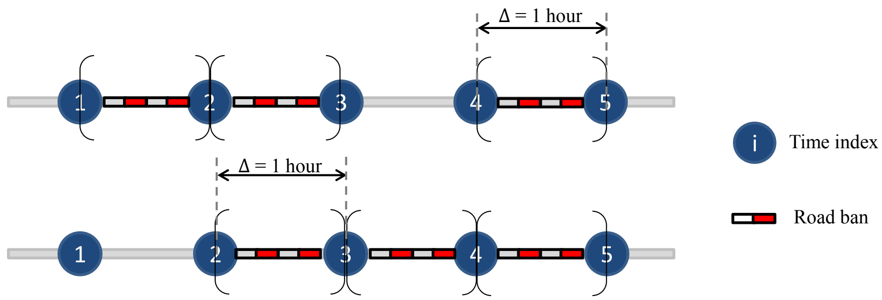


Figure 3: Example of a non-consecutive (top) and consecutive (bottom) closure of a road segment with Δ being the length of single time intervals, and $\overline{N} = 3$

- $\mathcal{L}$: Set of cycles of interest, indexed by l
- $[t_0, t_f]$: Time period of interest under each cycle l
- Θ_l: Set of discretized time periods corresponding to cycle l, indexed by t
- ω_{st}^{hl}: Binary variable taking 1, if section s is closed to hazmat type h shipment during cycle l at time t, 0 otherwise

Figure 3 illustrates the difference between the non-consecutive and consecutive closures of a road segment. Our definition of a consecutive section-time closure is as follows:

Definition 4 (Consecutive Closure)**.** A *consecutive section-time closure* decision of length $\overline{N}$ during an arbitrary cycle $l \in \mathcal{L}$ is comprised of a section $s \in \mathcal{S}$ and consecutive time indexes $t_i, t_{i+1}, \ldots, t_{i+\overline{N}-2}, t_{i+\overline{N}-1} \in \Theta_l$, and denotes closing all arc components of s during $[t_i, t_{i+\overline{N}}]$, where $t_{i+\overline{N}} = t_i + \overline{N}\Delta$ and $\overline{N} > 1$.

In order to enable the existing model to generate consecutive closures, we need an additional decision variable with the following notation:

$$z_{st}^{l} = \begin{cases} 1, & \text{if the closure block of section } s \text{ during cycle } l \text{ initiates at time } t \\ 0, & \text{otherwise} \end{cases}$$

for all $s \in \mathcal{S}, l \in \mathcal{L}, t \in \Theta_l$,

$$\delta_{sth}^{kcl} = \begin{cases} 1 & \text{if alternative } k \text{ of shipment } c \text{ becomes unavailable when section } s \text{ is closed to hazmat} \\ & \quad \text{type } h \text{ at time } t \text{ during cycle } l \\ 0 & \text{otherwise} \end{cases}$$

for all $c \in \mathcal{C}, k \in \mathcal{K}^c, s \in \mathcal{S}, l \in \mathcal{L}, t \in \Theta_l$, and $h = h(c)$. We provide the mathematical formulation of the time-dependent hazardous-network design problem with consecutive closures, TD-HNDP-II, as follows:

$$(\mathsf{TD\text{-}HNDP\text{-}II}): \qquad \min_{\omega_{st}^{hl}} Z(x_k^c) = \sum_{c\in\mathcal{C}}\sum_{k\in\mathcal{K}^c} r_k^c x_k^c + \epsilon \sum_{s\in\mathcal{S}}\sum_{l\in\mathcal{L}}\sum_{t\in\Theta}\sum_{h\in\mathcal{H}} \omega_{st}^{hl} \tag{12}$$

subject to

$$\sum_{h\in\mathcal{H}}\sum_{t\in\Theta_l} \omega_{st}^{hl} \le \overline{N}, \qquad \forall s\in\mathcal{S}, \forall l\in\mathcal{L} \tag{13}$$

$$\sum_{t\in\Theta_l} z_{st}^l \le 1, \qquad \forall s\in\mathcal{S}, \forall l\in\mathcal{L} \tag{14}$$

$$\sum_{h\in\mathcal{H}} \omega_{st}^{hl} - z_{st}^l = 0, \qquad \forall s\in\mathcal{S}, \forall l\in\mathcal{L}, t=1 \tag{15}$$

$$\sum_{h\in\mathcal{H}} \omega_{st}^{hl} \le z_{st}^l + \sum_{h\in\mathcal{H}} \omega_{st-1}^{hl}, \qquad \forall s\in\mathcal{S}, \forall l\in\mathcal{L}, t=2,3,\ldots|\Theta_l| \tag{16}$$

$$\sum_{k\in\mathcal{K}^c} x_k^c = 1, \qquad \forall c\in\mathcal{C} \tag{17}$$

$$y_k^c \le 1 - (\delta_{sth}^{kcl}\omega_{st}^{hl}), \forall c\in\mathcal{C}, \forall k\in\mathcal{K}^c, \forall s\in\mathcal{S}, \forall l\in\mathcal{L}, \forall t\in\Theta_l, h=h(c) \tag{18}$$

$$y_k^c \ge 1 - \sum_{s\in\mathcal{S}, l\in\mathcal{L}, t\in\Theta_l} \delta_{sth}^{kcl}\omega_{st}^{hl}, \quad \forall c\in\mathcal{C}, \forall k\in\mathcal{K}^c, h=h(c) \tag{19}$$

$$x_k^c \le y_k^c, \qquad \forall c\in\mathcal{C}, \forall k\in\mathcal{K}^c \tag{20}$$

$$x_k^c \ge y_k^c - \sum_{\ell=1}^{k-1} y_\ell^c, \qquad \forall c\in\mathcal{C}, \forall k\in\mathcal{K}^c \tag{21}$$

$$x_k^c, y_k^c \in \{0,1\}, \qquad \forall c\in\mathcal{C}, \forall k\in\mathcal{K}^c \tag{22}$$

$$\omega_{st}^{hl} \in \{0,1\}, \qquad \forall s\in\mathcal{S}, \forall l\in\mathcal{L}, \forall t\in\Theta_l, \forall h\in\mathcal{H} \tag{23}$$

$$z_{st}^l \in \{0,1\}, \qquad \forall s\in\mathcal{S}, \forall l\in\mathcal{L}, \forall t\in\Theta_l, . \tag{24}$$

The modeling principle in TD-HNDP-II is the set of alternatives for each shipment c, i.e., $\mathcal{K}^c$, with the alternatives being listed in increasing order of their corresponding travel cost incurred by the carriers. Later in Section 4.2 we clearly identify the alternatives of the carriers which are different from the original route-departure time alternatives of the TD-HNDP. The above model minimizes the total exposure to hazmat risk due to the minimum-cost routing/ scheduling decisions of the carriers. We also employ a fraction of the number of section-time closures during all cycles in the objective function for tie-breaking as well as eliminating unnecessary closures.

The TD-HNDP-II consists of all constraints of the TD-HNDP as well as additional constraints (14)–(16). In the above model, constraints (13) state that, irrespective of the hazmat type, a section cannot be closed for more than $\overline{N}$ time intervals during every cycle. Constraints (14) ensure that, for each section, at most one closure block exists during every cycle. Constraints (15) and (16) guarantee that the closures of a section during every cycle are sequential in time when $\overline{N}$ exceeds one. This is achieved via using auxiliary variables z_{st}^l keeping the start time index of a closure block during a cycle in constraints (15). The consecutive closures of the section during the corresponding cycle are then satisfied by constraints (16). While constraints (17)–(24) are similar to those of TD-HNDP, the availability of the alternatives for each shipment determined through constraints (18)–(19) is now impacted by the section-time closure decisions of the regulator within all cycles.

4.2 Stops En Route

We now extend the original TD-HNDP model to permit en route stopping which is beneficial for two reasons. First, according to Erkut and Alp (2007b), in integrated routing and scheduling with time-variant attributes, the incorporation of intermediate stops allow us to fully utilize the time-varying nature of the network. This is advantageous in two aspects: risk and cost. In fact, from the perspective of the regulator, a temporary road ban during rush hour can efficiently prevent a truck from entering a high-risk area of the network which reduces the hazmat exposure. On the resulting hazmat network, in the no-stop case, the hazmat truck is sent to a rather circuitous and longer alternate route. However, when stopping is allowed, taking time-of-day variations into account, the drivers can delay their departure from the origin node and/ or wait at the rest nodes to avoid congestion or a temporary road ban. This not only can further reduce the risk, but also is likely to improve the carriers' minimum-cost decisions.

Second, in the field of hazmat transportation, for sufficiently long trips, waits are mandated by the authorities to avoid drivers' fatigue which can increase the incident probability. For example, U.S. Department of Transportation (DoT) regulations effective since 2011 (Federal Motor Carrier Safety Administration, U.S. Department of Transportation, 2013) enforce the driver to be off-duty for a minimum of 10 hours after 14 hours of being on-duty (includes driving and rest stops). The driver may drive a total of 11 hours during the 14-hour on-duty period. A rest stop is mandatory after 8 hours of uninterrupted driving. The benefits of allowing stops en route are illustrated in Appendix G.

We introduce our new assumptions for allowing stops en route. Other than waiting at the origin node, the hazmat carriers may stop and wait at intermediate nodes along their route for resting and refueling. On the existing directed graph $\mathbb{G}(\mathcal{N}, \mathcal{A})$, we let $\Lambda \subset \mathcal{N}$ denote the set of rest areas, available to the drivers for stopping. For every shipment c, we assume there is an upper bound on the total duration of an alternative undertaken by the carriers, denoted by T_f, which includes not only the driving times but also the waiting times at the origin node and the rest areas. This parameter is usually identified by the hazmat trucking companies and prevents the drivers from taking frequent stops unnecessarily.

In order to consider the most realistic routing/ scheduling alternative which allows imposing the U.S. DoT regulations on the schedules, we adopt the restricted waiting and restricted driving time regulations as follows. The *uninterrupted driving time*—the driving time between two consecutive stops—cannot exceed a certain upper bound D. The *on-duty* period, which includes the driving times and the short breaks and excludes the waiting time at the origin, cannot exceed an upper bound W.

If the uninterrupted driving time is going to exceed D, *short break* at a rest node with time

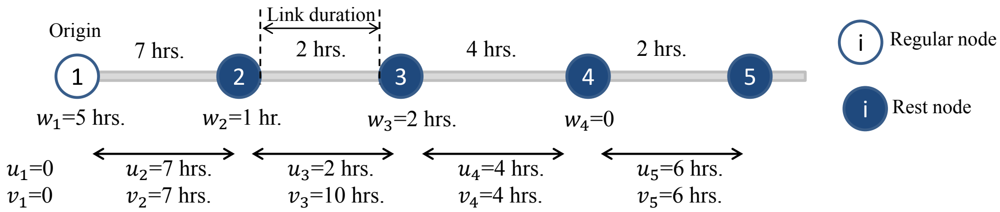


Figure 4: Example of a feasible routing/ scheduling alternative, with $D = 8\,\text{hr}$, $W = 12\,\text{hr}$, $L_{\text{sb}} = 1\,\text{hr}$, $U_{\text{sb}} = 2\,\text{hr}$, $L_{\text{lb}} = 2\,\text{hr}$, $U_{\text{lb}} = 5\,\text{hr}$

.

amount between L_{sb} and U_{sb} is mandated by the regulator. The driver is considered as on-duty during short breaks. If the on-duty period is going to exceed W, then a *long break* at a rest node of time amount between L_{lb} and U_{lb} is necessary due to regulations. The driver is considered as off-duty during long breaks.

We let u_i and v_i denote the uninterrupted driving time and the on-duty period duration on arrival at node i, respectively. Figure 4 illustrates an example of a feasible routing and scheduling alternative with stops along the way. A short break is required at node 2, otherwise the uninterrupted driving time upon arrival to node 3 would exceed D. Thus, when arriving at node 3, u_3 contains only the duration of link $(2,3)$, whereas v_3 includes the total duration of driving as well as the short break (w_1) excluding the waiting time at the origin. If no stop is taken at node 3, the on-duty period would be more than W upon arrival to node 4. Therefore, a long break within the permissible range is required at node 3 (w_2). After a long break, both u and v variables are set to zero and the process continues.

Consequently, with the above assumptions, in the current model, not only should an alternative identify the route and the departure time from the origin, it should also determine the location of the stop areas and the duration of the stops. This requires modifying the original Definition 2 of an alternative as follows, recalling that $\mathcal{P}^c$ corresponds to the set of all routing options between $o(c)$ and $d(c)$ for shipment c.

Definition 5 (Route-Time Alternative for TD-HNDP-II)**.** For a shipment c from the origin $o(c)$ to the destination $d(c)$, and the earliest departure time from the origin node as t_0^c where $t_0^c \in \Theta_l, l \in \mathcal{L}$, a *route-time alternative* $k \in \mathcal{K}^c$ is characterized as the combination of a route $p \in \mathcal{P}^c$, a departure time from origin, $t \geq t_0^c$, a set of stop areas $R_k \subset \Lambda$ on route p, as well as their corresponding waiting times $w_i, \forall i \in R_k$, such that the total duration of the alternative (includes the driving times as well as the waiting times at the origin and the stop areas) in no more than T_f.

Note that the two attributes associated with an alternative k, namely the risk and the cost, are calculated with a procedure analogous to the one for the original TD-HNDP. For calculation of the risk, however, the probability of an accident is assumed to be zero during a stop. This is a reasonable assumption since both the accident probabilities and consequences when the vehicle is not moving are likely to be considerably lower than the case where the vehicle is moving. Therefore, the risk of an alternative is computed by accumulating the risk of the road links traversed on the route at their corresponding entrance times, ignoring the risk at the stop locations. In the following, we develop an algorithm to generate alternatives defined as in Definition 5.

4.3 Solution Methodology for the Extended Model

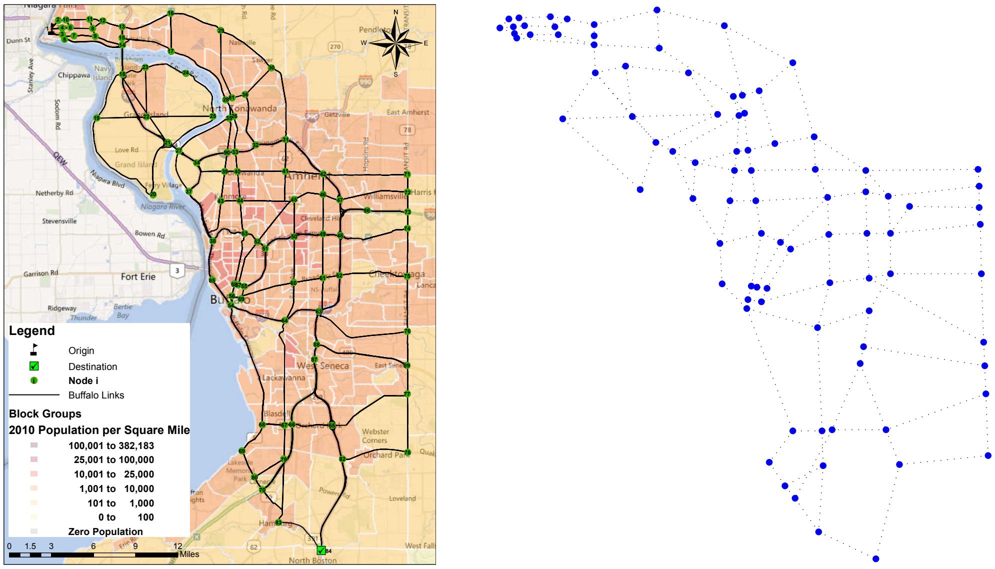
The difference between the TD-HNDP and TD-HNDP-II is that (i) the latter introduces additional constraints to the problem to account for consecutive time-based section closures, and (ii) for a shipment c, an alternative $k \in \mathcal{K}^c$ is now comprised of a route and schedules of departures from the origin node as well as the intermediate rest nodes. Our observations from numerical experiments reveal that, with minor changes in the algorithm, the column-generation-based heuristic proposed for the TD-HNDP can well accommodate both modifications. The total enumeration procedure is shown to be computationally extremely demanding and infeasible for any but very small test problems. Nonetheless, when the optimal solution is obtainable by the total enumeration, it is used to verify the results of our heuristic. Our experiments illustrate that the heuristic guarantees to be promising as a valid solution methodology that can generate high quality solutions, optimal solutions for our instances, in significantly reduced amount of time. Therefore, for attaining the time-dependent section closure policies in TD-HNDP-II, we propose using a column-generation-based procedure analogous to the one proposed for the original TD-HNDP.

A key change is that we need to solve time-dependent elementary shortest-path problems with resource constraints and intermediate stops for the pricing sub-problems in the algorithm. In this purpose, we develop a label setting algorithm based on the dynamic programming approach of Feillet et al. (2004). The details are found in Appendix E. With the new label setting algorithm on hand, we make the following changes:

1. We let the linear programming relaxation of the TD-HNDP-II, defined using a subset of the feasible alternatives, i.e., route and schedules of waitings, which is denoted by $\overline{\mathcal{K}}^c$ for every shipment c, construct the RMP.

2. To obtain the minimum-cost decisions of the carriers either in the unregulated network, i.e., generation of the initial alternatives, or in response to the section-time closure policy in every iteration, i.e., solving the ASP, we apply the label setting algorithm described in Appendix E, with time-varying travel times being the arc impedances.

3. The PSP is a time-dependent elementary shortest-path problem with resource constraints and intermediate stops, where the time-varying risk attributes are revised using a procedure analogous to (10) and (11). We adapt the label setting algorithm of Appendix E to solve the PSPs in each iteration, enabling us to find a set of good alternatives with *negative* reduced costs for every shipment which are also elementary. This requires modifications to the definitions of the label and the dominance rule. The details of this procedure are delegated to Appendix F.



(a) Actual Map

(b) Simplified Road Network

Figure 5: The Buffalo, NY Network (Toumazis and Kwon, 2013)

5 Numerical Experiments

The goal of our numerical experiments is three-fold: (i) to explore the viability of the algorithms proposed for solving TD-HNDP and TD-HNDP-II, (ii) to compare the cost-risk trade off of the solutions provided by imposing different control levels from the regulator, and (iii) to demonstrate that the TD-HNDP-II can produce design decisions that follow realistic policy and driving restrictions. For this purpose, we provide an application of our proposed methodology in Buffalo, NY. We first present a description of the problem data used as a basis of our analysis, followed by a summary of our experiments and findings.

5.1 The Buffalo Network Data Set

The time-varying network over which our experiments are conducted is a portion of the road network of the city of Buffalo, NY, used in Toumazis and Kwon (2013). The network is comprised of 90 nodes and 149 arcs as shown in Figure 5. Each arc has four attributes: population density, time-dependent regular traffic flow, time-dependent travel duration, and hazmat accident probability. The information on the census subdivisions of city of Buffalo was used to obtain the spatial distribution of population and estimate the population density of each road link. The time-dependent attributes were constructed for a study period of 24 hours, namely between 12:00 a.m. and 11:59 p.m., which has been divided into 1-hour time intervals ($\Delta = 1\,\text{hour}$).

We consider 20 sections, comprising of 32 links, within the region of interest which are available for closure on an hourly basis. That is, closing a section at time t denotes imposing curfews on all its corresponding links from t to $t+1$. The length of the road segments in the original network of

Buffalo is too short to properly demonstrate the capabilities of our model as it is possible to travel between the two farthest nodes of the network in less than an hour. Hence, we scale the original network data by multiplying the lengths of all segments by a factor of ten. Although the realism of the results is somewhat reduced, this provides us useful insights on the problems and methods.

Finally, since the data on the actual trips taken by the hazmat carriers through U.S. highways is neither sufficient nor reliable, we randomly generate multiple hazmat shipments including origin, destination, the number of trucks used, as well as their value of time, PDT, PAT, and the penalty parameters assuming that every shipment carries a distinct hazmat type with an associated radius of contamination.

We have implemented our algorithm in Matlab 2014b using CPLEX 12.6.1 for solving LP or MILP problems. All experiments are performed on an Intel 2.40 GHz PC with 32.0 GB of RAM.

5.2 The Base Model

In this section, we test the base model, TD-HNDP, with the following settings. First, we used $\epsilon = 0.0001$. Second, we consider one type of hazmat, that is, ω_{st}^h is replaced by ω_{st}, for simplicity. Third, the network regulation policy obtained by the column-generation-based heuristic is compared with those obtained under the following three scenarios:

– *total enumeration*: we apply an enumerative procedure to explicitly generate all feasible route-time alternatives for each shipment. After constructing the comprehensive $\mathcal{K}^c, \forall c \in \mathcal{C}$, the optimal solution of TD-HNDP is obtained using CPLEX.

– *over-regulated model*: This corresponds to the ideal scenario of the regulator wherein the risk can be minimized to its lowest possible value via direct control over the route decision of carriers. We solve a time-dependent shortest path problem (TDSPP) with the hazmat risk being the travel cost.

– *unregulated model*: This corresponds to the case when the regulator takes no action, which is the most preferable scenario to the carriers. We solve a TDSPP with the travel cost.

We first present the results of our tests under different number of shipments as in Table 2 with number of sections and the closure frequency set to 3 and 1, respectively. We note that the column-generation-based algorithm produces the same total risk of the network and the same associated average cost imposed to each truck as the total enumeration scenario. That is, the heuristic approach produces an optimal solution for all cases considered. Comparing the number of alternatives generated under the two algorithms, however, indicates that, for each problem instance, much fewer columns are considered in the TD-HNDP under the column-generation-based heuristic in order to attain the solution. This implies to the ability of the proposed heuristic to implicitly account for the columns in the optimal basis through PSP and ASP. Consequently, the heuristic algorithm takes much less CPU time.

Table 2: Solution of the TD-HNDP: performance of the column-generation-based heuristic vs. the total enumeration under different number of shipments. CPU time is in seconds, $|\mathcal{S}| = 3$, and $\overline{N} = 1$.

# of shipments	Column Generation Based Heuristic					Total Enumeration			
	Total Risk[a]	Avg. Cost	No. of Iterations	No. of Alt.[b]	CPU Time	Total Risk[a]	Avg. Cost	No. of Alt.[b]	CPU Time
2	60.81	53.67	3	68	3.00	60.81	53.67	98	338.55
5	163.91	93.43	5	108	10.50	163.91	93.43	243	402.79
7	194.71	98.27	5	119	12.06	194.71	98.27	266	441.40
10	225.55	91.09	5	144	16.38	225.55	91.09	351	4365.56
12	283.38	94.46	5	149	17.61	283.38	94.46	356	4386.26
15	312.83	88.83	6	162	20.36	312.83	88.83	398	4652.32
20	351.47	79.23	6	239	32.73	351.47	79.23	546	8248.69

[a] Refers to the total transport risk due to carriers' minimum-cost decisions after the implementation of the design policy.
[b] Denotes the total number of alternatives generated under the corresponding algorithm, i.e., $\sum_{c\in\mathcal{C}} |\mathcal{K}^c|$

Table 3: The compromise solutions each corresponding to a different number of sections considered for closure in the TD-HNDP, with $|\mathcal{C}| = 7$, and $\overline{N} = 1$.

% Change of [a]	Number of sections included in TD-HNDP							Over-regulated
	3	4	6	8	10	15	20	scenario
Total Risk	-5.06	-5.06	-5.43	-12.12	-12.94	-27.56	-28.45	-41.76
Cost	16.87	16.87	17.24	22.08	24.46	28.16	27.35	33.90

[a] Refers to the % change over the *unregulated scenario* with total risk and average cost being 205.09 and 84.08, respectively.

To illustrate the flexibility of the TD-HNDP framework in determining compromise solutions between the regulator and the carriers, we carried out additional experiments by imposing various intervention levels provided by changing either the 'number of sections $|\mathcal{S}|$' or the 'closure frequency of every section, $\overline{N}$, in constraint (2)'. For instance, we solved the TD-HNDP when the number of shipments and the closure frequency are set to 7 and 1, respectively, whereas, the number of sections considered for closure varies between 3 and 20. Table 3 represents the percentage change in total risk and average travel cost for each case under the TD-HNDP as well as the over-regulated network when compared with the unregulated scenario.

Each column in Tables 3 corresponds to an alternative regulatory scheme to the network design problem with a certain compromise between the regulator and the carriers in terms of the associated transport risk and economic viability. In fact, the level of government intervention increases as one gradually moves from the left-most schemes to the over-regulated cases. To better observe this, we present a trade-off curve in Figure 6 by changing $|\mathcal{S}|$. The total risk exposure and the average cost values corresponding to each problem instance are normalized. The left-most point in Figure 6 is associated with the unregulated scenario, whereas the right-most point represents the

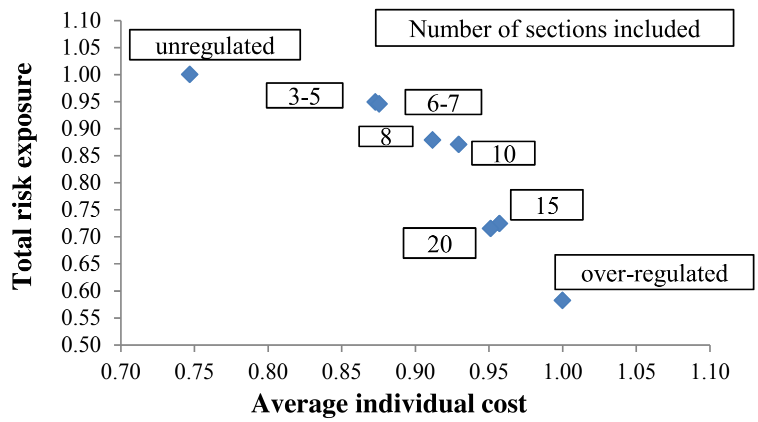


Figure 6: The risk-cost trade-off for alternative designs, with $|\mathcal{C}| = 7$, and $\overline{N} = 1$

desired scenario of the regulator. Every other point within these two extreme points provides useful information regarding the status of the cost-risk trade-off under each alternative solution to the TD-HNDP, thus facilitating healthy consultation between the two parties during the policy design process. We conclude that the proposed framework can help engage hazmat carriers in the design process by varying the intervention level of the regulator until a mutually acceptable section-time closure decision is reached.

5.3 The Extended Model

In this section, we compare the original TD-HNDP model with the consecutive closure scenario as well as the stopping scenario. We use the Buffalo netowrk for a time period of 48 hours, consisting of 2 cycles (days) of length 24 hours. We assume that the time period of interest is divided into intervals of length 15-minute ($\Delta = 15$ minute). For this purpose, provided by the time-variant arc attributes for 5-minute periods, the arc attributes associated with 15-minute intervals are constructed by averaging over the corresponding 5-minute periods. Although the resulting problem would require more computational effort than the case with $\Delta = 1$ hour, it provides us with a more accurate and realistic numerical example.

We assume 7 sections, comprising of 7 links, are available to the regulator for imposing time-based curfews. In particular, $\mathcal{S} = \{(14, 18), (82, 84), (37, 38), (40, 44), (21, 27), (48, 62), (87, 65)\}$. Despite we use 15-minute time intervals, we let the length of every time-based closure be an hour, i.e., four 15-minute time intervals, for practical reasons. The maximum closure frequency of every section during every cycle, i.e., $\overline{N}$, is set to 4, implying that a section cannot be closed for more than four hours on a certain day. The rest areas are assumed at the following nodes: $\Lambda = \{8, 18, 22, 29, 34, 48, 54, 63, 65, 68, 74, 76, 82\}$. We let the waiting time at each node be between zero and 4 hours.

The upper bound on the uninterrupted driving time is $D = 5$ hours, mandating a short break with $L_{\text{sb}} = 15$ minutes and $U_{\text{sb}} = 1$ hour. The maximum on-duty period (including the driving times and the short breaks between two long breaks) cannot exceed $W = 9$ hours, which necessitates

Table 4: Network characteristics under two alternative Designs I and II to study the impact of consecutive road-closures. Values in brackets indicate percent change over the unregulated network for the corresponding criterion.

Criterion	Unit	Unregulated Network[a]	Alternative Network Design: Design I	Design II
Total Risk[b]	truck-people	13.11	10.73 [-18.1%]	11.17[-14.8%]
Avg. Cost	$	82.45	84.03 [1.9%]	83.63[1.4%]
Avg. Total trip time [c]	hour	15.42	17 [10.2%]	16.6[7.6%]
Avg. Total driving time [d]	hour	15.42	17 [10.2%]	16.6[7.6%]
Avg. Total waiting time [e]	hour	0.00	0.00	0.00
CPU time	hour	—	1.08	1.08

[a] Corresponds to the minimum-cost alternatives of the carriers when waiting is only allowed at the origin.
[b] Indicates the accumulated risk of all shipments induced to the network.
[c] Includes the driving time and the waiting time at the origin.
[d] Implies to the uninterrupted driving time from the origin to the destination.
[e] Refers to the waiting at the origin node.

a long break with L_{lb} and U_{lb} being 1 and 4 hours, respectively.

While the total enumeration approach with CPLEX failed to deliver any answer within a day, the proposed heuristic approach provided an answer within 1 to 3 hours for all numerical experiments we performed. Note that the computational time for the extended model is significantly increased compared to the time for the base model. This is because of the large number of possible combinations in generating alternatives with stops en route.

We consider three designs: Design I. non-consecutive closures without intermediate stops; Design II. consecutive closures without intermediate stops; and Design III. consecutive closures with intermediate stops.

5.3.1 Non-Consecutive (Design I) vs. Consecutive Closures (Design II)

To demonstrate the capability of the TD-HNDP-II in generating consecutive time-based road closures, as well as to realize the impact of such policies on the network characteristics, we compare the network regulated under Design I with the one obtained under Design II. Table 4 summarizes the implications of these alternative regulations, as well as the corresponding unregulated scenario for a valid judgment.

As one expects, both Designs I and II result in reducing the total risk at the expense of increasing the average travel cost and average total trip time of the carriers, over the unregulated scenario, see Table 4. From the theoretical point of view, however, it is conceivable that the optimal value of the objective function, i.e., the total risk, under Design II is likely to deteriorate due to the inclusion of constraints (14)–(16). In fact, allowing to impose only consecutive road-closures implies to the regulator's risk-mitigation ability being reduced. Therefore, the risk reduction under Design II is 4% less than the one obtained under Design I, which clearly involves with less increase in the carriers' cost compared to their minimum-cost alternatives. Nevertheless, it is important to note that not

Table 5: Network characteristics under regulatory schemes (II) and (III) to investigate the impact of stopping en route. Values in brackets indicate percent change over the corresponding unregulated network for each criterion.

Criterion	Unit	Unregulated Network		Alternative Network Design	
		No-stopping[a]	Stopping[b]	Design II	Design III
Total Risk[c]	truck-people	13.11	14.71	11.17[-14.8%]	10.06[-31.6%]
Avg. Cost	$	82.45	88.56	83.63[1.4%]	97.37[9.9%]
Avg. Total trip time[d]	hour	15.42	19.54	16.6[7.6%]	24.18[23.7%]
Avg. Total driving time[e]	hour	15.42	15.48	16.6[7.6%]	15.23[-1.6%]
Avg. Total waiting time[f]	hour	0.00	4.07	0.00	8.95[119.9%]
CPU time	hour	—	—	1.08	3.68

[a] Refers to the case where waiting is only allowed at the origin.
[b] Refers to the case where waiting is allowed at the origin and the intermediate nodes.
[c] Indicates the accumulated risk of all shipments induced to the network.
[d] Includes all the driving times, and the waiting times at the origin and the intermediate nodes (if allowed).
[e] Implies to the accumulated driving times from origin to the destination.
[f] Implies to the accumulated waiting times at the origin and the intermediate nodes (if allowed).

only Design II still does make a considerable change in terms of risk, it also generates more practical design policies requiring less expenditures for implementation.

It is evident that, from Design I to Design II, the issue regarding the sparse road-closure policy is resolved. Further, our observations in the computational experiments do not prove any significant additional computation effort to the problem for handling the new constraints (14)–(16). Note the identical CPU times required under Designs I and II in Table 4. Nevertheless, one remaining issue corresponds to the unreasonably long driving times of the carriers' trips. In particular, we note that the uninterrupted driving times of the alternatives used by the carriers under Design II range between 10 to 24 hours which are impractical and do not comply with the regulations. Therefore, one needs to further refine the generated design policy by accounting for stops in the minimum-cost alternatives of the carriers which constitutes the third version of the problem, i.e., Design III.

5.3.2 No-Stopping (Design II) vs. Stopping En Route (Design III)

Note that Designs II and III are common in obtaining consecutive road-closures, but different in disallowing and allowing stopping en route. Table 5 summarizes the network characteristics of the regulatory schemes (II) and (III) as well as their corresponding unregulated scenarios for a better comparison.

We first compare the characteristics of the unregulated networks for no-stopping and stopping scenarios according to our results in Table 5. We observe that despite stopping en route can reduce the risk implications of the minimum-cost alternatives over the no-stopping scenario for some of the shipments, in our example, the total risk induced to the network is increased due to mandatory rests along the way. In fact, under the unregulated scenario and when stopping is allowed, the carriers choice of waiting at a node has two reasons: avoiding the congestion, and complying with the regulations. While the former is most likely beneficial from both risk- and cost-minimization perspectives, the latter might face the drivers to higher-exposure time periods compared with no-stopping scenario. Furthermore, while the average driving times are almost identical between the no-stopping and stopping scenarios, the average total trip time is increased due to the inclusion of the inevitable waiting times along the route. This indeed explains the excess average cost of the stopping scenario.

We now analyze the impact of the en route stops in designing a hazmat network. It is evident, from Table 5, that Design III results in the generation of routes and schedules of the carriers that involve much lower levels of risk exposure than those where waiting is not allowed—a further reduction of 16.8% is achieved in risk compared to Design II. Although, for such a reduction, the carriers would clearly experience longer trip durations under Design III, i.e., 23.7% vs. 7.6% increase over the corresponding unregulated scenarios, one can notice that, on the average, they require less driving times to complete a trip. In fact, the excess trip time is mainly due to the waiting times which are either mandated by the authorities for resting and consequently are inevitable, or are imposed by the regulator through road closure policy for risk-mitigation. Furthermore, recall that Design II does not account for the realistic behavior of the carriers in decision-making by disregarding the necessary stops along the way. Therefore, it is likely that the regulator would not be able to achieve the prescribed reduction in the risk upon implementation. In contrast, Design III is produced under the consideration of en route stops in routing and scheduling decisions of the carriers as well as the complex restrictions mandated by the authorities. In effect, such alternatives are more likely to be undertaken by the carriers in practice, and thereby guarantee the effectiveness of the design policy. However, the resulting problem under Design III is more complicated and requires more computational effort.

From Design II to Design III, one can guarantee that the routing and scheduling decisions anticipated to be used by the carriers are indeed practical. In fact, not only can Design III outperform Design II in mitigating hazmat risk by taking full advantage of the time-varying network via intermediate stops, it also appears to be promising in terms of the ability of implementing the prescribed solution by the regulator. However, the compensation for obtaining such an improved solution in Design III is the increased complexity.

6 Conclusion and Future Research

In this paper, we consider the time-dependent hazardous-network design problem, and expand the effectiveness and the applicability of the generated design policies by incorporating two refinements: the time-based road closure polices are restricted to be consecutive, and waiting at intermediate nodes is allowed in generating routing and scheduling decisions of the carriers. We propose a column-generation-based heuristic to solve the problem where each column represents a route-time alternative. An auxiliary subproblem is embedded in the algorithm to generate more columns from the carriers' route-time choices to the design policies; thus reinforcing the performance of the heuristic. Our numerical tests on viability of the column-generation-based heuristic verifies its solution quality and computational efficiency. Additional experiments illustrate the capability of our proposed framework in producing mutually acceptable solutions for the regulator and the carriers.

We study three versions of the problem by gradually incorporating the two refinements. We observe that the risk-mitigation ability of the regulator deteriorates when the road closures are restricted to be consecutive. This, however, is at the benefit of reducing the efforts and expenditures for implementation of the policy. Taking advantage of the en route stops, our results indicate that, to prevent a truck from entering a high-risk area/time period, the regulator can determine more efficient time-based road-closure polices some of which are considered as ineffective when stopping is not allowed. On the other hand, the stops en route provide a richer decision-making environment for the carriers, and thereby are likely to improve their routing and scheduling choices from the cost and the transport risk perspectives. This version is the most realistic scenario which allows imposing complex restrictions on the carriers' trips, and consequently assures the regulations during the policy design process.

Our observations from computational experiments indicate that our column-generation-based algorithm can solve the most realistic scenario of the problem within reasonable computational time. However, when stops are allowed, the pricing sub problems requires much more effort to execute, especially when the length of each time period is small, due to exponentially many routing-scheduling alternatives can be identified by incorporating stops and different durations of waiting. Therefore, we believe that developing heuristic algorithms for generating routing-scheduling decisions of the carriers with intermediate stops is more efficient when the networks are significantly larger.

We assume the accident probability of a parked hazmat vehicle be zero in our model. That is, there is no risk associated with nodes of the network. This may not be true as, when the hazmat truck is not moving, there is still a chance of release or explosion of its contents due to crashing, insecure parking conditions, etc. Therefore, an interesting extension to this work would be to consider node-dependent risk attributes where the accident probability of stopping is a function of the duration of stops.

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Appendices

A The Discretization Procedure

We assume that the time period of interest, i.e., $[t_0, t_f]$ is discretized into small intervals. Let

$$\Theta = \{t_0, t_0 + \Delta, t_0 + 2\Delta, \dots, t_0 + M\Delta\}$$

denote the discrete time index set with t_0 being the earliest possible departure time from any node in the network. Constants Δ and M are both user-defined, and denote a small time interval during which some perceptible change in traffic conditions may occur, and a large integer such that the interval from t_0 to $t_0 + M\Delta$ covers the desirable time period of study, respectively. Further, we let $\tau_{ij}(t)$ be the non-negative time required to travel from i to j when departing from node i at time t. Note that $\tau_{ij}(t)$ is a real-valued function defined for every $t \in \Theta$, and satisfies the FIFO property. For simplicity, our model considers $\tau_{ij}(t)$ for $t > t_0 + M\Delta$ be infinite eliminating all route-time alternatives with arrival time to a destination node beyond $t_0 + M\Delta$. This is a reasonable assumption as we focus on regulating hazmat shipments which not only emerge but also terminate within the time period of interest. Nonetheless, it is not a restrictive assumption, and one can simply account for possibility of arriving at a destination node after $t_0 + M\Delta$ by assuming that $\tau_{ij}(t)$ for $t > t_0 + M\Delta$ is constant and equal to $\tau_{ij}(t_0 + M\Delta)$.

When computing τ_{ij}, it is possible that the actual departure time from node i is not included in set Θ, i.e., $t' \notin \Theta$. To avoid ambiguity, it is assumed that

$$\tau_{ij}(t') = \tau_{ij}(t_0 + m\Delta), \quad \forall t' \in (t_0 + m\Delta, t_0 + (m+1)\Delta)$$

However, when elements of Θ are all integer values, we simply round t' to its nearest integer. That is, $\tau_{ij}(t') = \tau_{ij}([t'])$.

In addition, we let $f_{ij}(t)$ be the non-negative regular traffic flow on arc (i, j) when departing from node i at time t; $f_{ij}(t)$ is defined for every $t \in \Theta$, and is computed for $t' \notin \Theta$ similar to $\tau_{ij}(t')$. Therefore, both the travel time and regular traffic can take discrete values.

B The Risk of an Alternative

In order to compute the hazmat risk exposure of an alternative, we first let the risk associated with a truck carrying hazmat type h through arc (i, j) when departing from node i at time t be denoted by

$$r_{ij}^h(t) = (\rho_{ij}^h + f_{ij}(t))\mathrm{Pr}_{ij}, \tag{25}$$

where ρ_{ij}^h is the number of people exposed on arc (i, j) when hazmat type h is carried, and it is assumed to be the number of people living within the circular area of radius λ_h centered at the accident location. That is, $\rho_{ij}^h = (\pi\lambda_h^2)\rho_{ij}$.

It is notable that the accident rate usually depends on the road type and congestion as well as the time (e.g., whether daytime or nighttime) the road is crossed. Nonetheless, for simplicity of our model, we assume the hazmat accident probability be only dependent on the road type, and represented by a constant usually in the range of $[10^{-8}, 10^{-6}]$ per mile traveled (Harwood et al., 1993).

Equipped with the definition of the link risk, we are now ready to calculate the risk associated with an alternative. For shipment c, with n^c number of trucks carrying hazmat type $h(c)$ from $o(c)$ to $d(c)$, and t_0^c denoting the earliest possible departure time from $o(c)$, we let the hazmat risk exposure associated with alternative $k \in \mathcal{K}^c$, i.e., r_k^c, be calculated as below:

$$r_k^c = \sum_{(i,j)\in\mathcal{A}^p} n^c r_{ij}^{h(c)}(\phi_{ij}^p(t)) = \sum_{(i,j)\in\mathcal{A}^p} n^c(\rho_{ij}^{h(c)} + f_{ij}(\phi_{ij}^p(t)))\mathrm{Pr}_{ij}, \tag{26}$$

with $\mathcal{A}^p \subset \mathcal{A}$ being the set of road links traversed when using path p, and $\phi_{ij}^p(t)$ calculating the time moment at which the shipment enters link (i,j) on path p when departing from the origin at time t. We can express:

$$\phi_{i_1j_1}^p(t) = t \tag{27}$$
$$\phi_{i_2j_2}^p(t) = t + \tau_{i_1j_1}(\phi_{i_1j_1}^p(t)) = t + \tau_{i_1j_1}(t) \tag{28}$$
$$\phi_{i_3j_3}^p(t) = t + \tau_{i_2j_2}(\phi_{i_2j_2}^p(t)) = t + \tau_{i_2j_2}(t + \tau_{i_1j_1(t)}) \tag{29}$$
$$\vdots$$

C The Cost of an Alternative

The hazmat trucking companies are usually in collaboration with the origin and destination nodes at the time of pick up and delivery, respectively, based upon an agreement. Hence, it is highly probable that there are fees charged to the trucking companies when deviating from the prescribed pick up/ delivery times. Therefore, in addition to the travel time of the undertaken route, the times of the departure from origin and the arrival to destination play an important role in characterizing the travel cost of an alternative. Table 6 describes the notation.

For shipment c, let g_k^c denote the time-dependent travel time associated with traveling along route p when departing from the origin $o(c)$ at time t, i.e., alternative k. Accordingly, the travel cost of this alternative is identified using a function of the following form:

$$u_k^c = \eta^c g_k^c + \mu^c|t - \kappa^c| + \nu^c|(t + g_k^c) - \sigma^c| \tag{30}$$

with $(t + g_k^c)$ being the arrival time to the destination. Ideally, shipment c prefers to depart from the origin at its earliest possible departure time implying that $\kappa^c = t_0^c$. For analytical tractability, it is also assumed that $\eta^c \geq \nu^c$. This assumption allows the travel cost of an alternative k, u_k^c, be a nondecreasing function of its travel time, g_k^c, for a fixed departure time t. That is, among all the

Table 6: Notation used to define the cost of an alternative

Notation	Description
η^c	The value of time for the carriers of shipment c [\$/hr] [1]
κ^c	Preferred Departure Time (PDT) for the carriers of shipment c [hr] [2]
σ^c	Preferred Arrival Time (PAT) for the carriers of shipment c [hr] [3]
μ^c	Penalty charged to shipment c for deviating from PDT [\$/hr] [2]
ν^c	Penalty charged to shipment c for deviating from PAT [\$/hr] [3]
g_k^c	Time-dependent travel time experienced by each truck of shipment c when choosing alternative k [hr]
u_k^c	Time-dependent travel cost incurred per truck of shipment c when choosing alternative k [\$]

[1] Refers to the value of time of the corresponding hazmat trucking company.
[2] Specified through negotiation between the origin node company and the hazmat trucking company.
[3] Specified through negotiation between the destination node company and the hazmat trucking company.

alternatives departing from the origin at time t, the alternative with the shortest travel time also denotes the least-cost alternative. This has been further clarified in Section 3.1.

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Supplemental Material

These appendices may be published as part of online supplemental material.

D Comparing the TD-HNDP Model with the Static Model

We show that how time-dependent policies can improve the hazmat network design, comparing with the static design policies. For this purpose, we present an application of the proposed framework to determine the road segments in the City of Sioux Falls, SD that should be closed to hazmat trucks on a timely basis. We first describe the characteristics of the network in Section D.1, and then summarize the results of our analysis in Section D.2. Furthermore, in Section D.3, the case study is used to present a convincing argument that taking the time-based variations of the network parameters into account, the time-dependent network design framework provides a richer decision-making environment than the static network design, thereby is more likely to generate mutually acceptable scenarios for the regulator and the carriers.

D.1 The Data Set

As a basis for our case study, we use the Sioux Falls, SD road network, consisting of 24 nodes, 76 arcs, and 24 population centers. The website of Transportation Network Test Problems (Bar-Gera, 2013), which constitutes our primary source of data, provides the network, the trip table, and static arc attributes including capacity, length and free flow travel time. The Census site information enables us to generate the population density around each road segment, and estimate the at-risk population using the λ-neighborhood concept. The hazmat accident probability, associated with each arc, is computed according to the methodology described in Toumazis and Kwon (2013). A simulation-based dynamic traffic assignment software provided by DynusT is employed to construct the time-dependent arc attributes. To this end, we let the time period of study to hold between 6:00 a.m. and 9:59 p.m., which has been divided into 1-hour time intervals. The original travel demands between the 552 OD pairs in this network serve as the base estimation of the hourly OD demand parameters. However, these values are adjusted by a factor of 1.2 for the intervals of 8 a.m. to 11 a.m. and 5 p.m. to 8 p.m. to capture the morning and evening road congestion. DynusT is then used to solve the dynamic traffic assignment problem pertaining to this data set, providing us with the accumulated hourly traffic volume for each arc. Finally, the corresponding time-dependent arc travel times are calculated using the BPR function.

For our testing, we assume that the regulator has the authority to close at most 4 sections, consisting of 8 links, with the length of every time-based closure being an hour. The latter is a reasonable assumption since curfews are more often imposed on an hourly basis in practical applications. Nevertheless, it is not restrictive as multiple hours of closure can be simply imposed with $\overline{N} > 1$. The map of the region, the underlying road network, as well as the sections are displayed in Figure D.1.

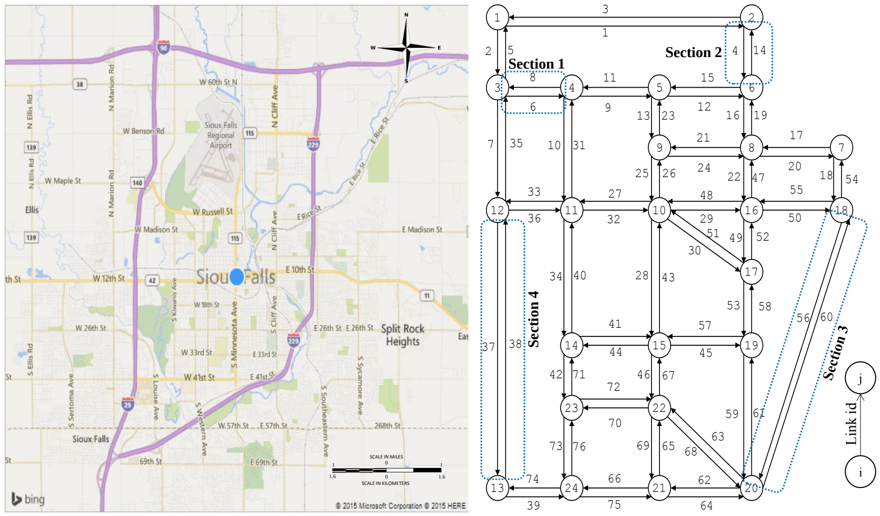


Figure D.1: Sioux Falls Network

For tractability, the case study analysis involves two hazmat shipments within this region with the first one being a truck from node 1 to 20 emerging at 10 a.m., and the second being a truck from node 7 to 13 emerging at 7 a.m. Since the purpose is to illustrate the use of TD-HNDP in identifying compromise solutions between the regulator and the carriers in a realistic setting, but not to do the site specific data collection, we have generated plausible estimates of shipment specific parameters including VoT, PDT, PAT, penalty parameters, as well as radius of contamination.

D.2 The Results of the Analysis

The column-generation-based heuristic is used to solve the TD-HNDP with $\epsilon = 10^{-5}$. Similar to Section 5, we use Matlab 2014b and CPLEX 12.6.1 to implement our model, and perform the tests on an Intel 2.40 GHz computer. We analyze the characteristics of four different solutions to the TD-HNDP for: (i) *2 sections* & $\overline{N} = 1$, (ii) *2 sections* & $\overline{N} = 3$, (iii) *4 sections* & $\overline{N} = 1$, and (iv) *4 sections* & $\overline{N} = 3$, as well as the unregulated and over-regulated scenarios, each corresponding to an alternative regulatory scheme. Table D.1 summarizes the implications of these alternative regulations. The graphical representation of the proposed design policies and the route-time decisions undertaken by the carriers are also depicted in Figure D.2.

In Table D.1 and Figure D.2, the regulatory schemes are sorted according to the regulator's intervention level. Evidently, the total risk decreases and the average cost increases as one moves towards the bottom. Figures D.2(a) and D.2(f) display the cost-minimizing alternatives of the carriers and the minimum-risk alternatives of the regulator, respectively. Note that under D.2(f), the regulator assigns a rather circuitous route to shipment 1, whereas a 4-hour delay in departure time is designated for shipment 2. Despite the 65% reduction in total risk, these alternatives are on average 184% more costly than the carriers' preferences, which is very undesirable from a shipper's perspective. However, we demonstrate that it is possible to obtain low-risk alternatives that are also acceptable to the carriers by varying the regulator's legislative power in TD-HNDP.

Table D.1: Implications of alternative regulatory schemes

Scenario (sections & $\overline{N}$)	Total Risk [a]	Average Travel Cost \$	Shipment 1 Risk [b]	Shipment 1 Travel Cost \$	Shipment 1 Travel Time *hr.*	Shipment 2 Risk [b]	Shipment 2 Travel Cost \$	Shipment 2 Travel Time *hr.*
Unregulated	503.93	173.14	180.10	190.06	3.25	323.83	156.22	1.41
(i) (2 & 1)	479.46	173.68	155.63	191.14	3.56	323.83	156.22	1.41
(ii) (2 & 3)	337.69	181.06	155.63	191.14	3.56	182.07	170.97	1.77
(iii) (4 & 1)	336.84	209.09	154.78	247.20	3.34	182.07	170.97	1.77
(iv) (4 & 3)	279.95	213.76	155.63	191.14	3.56	124.32	236.39	2.66
Over-regulated	176.86	491.50	91.51	488.88	7.61	85.35	494.13	4.43

[a] Indicates the accumulated risk induced to the network, in *truck-people* unit and (10^{-4}) scale,
[b] Refers to the risk value induced to the network by the route-time decisions of the corresponding carriers, in *truck-people* unit and (10^{-4}) scale.

From Figure D.2(b) and Table D.1, one observes that, by closing section 2 during $[11am, 12pm]$, shipment 1 has to take a different alternative than the minimum-cost one (its second best), while shipment 2 can still use its most preferred alternative. As a result, the regulator achieves a nearly to 5% reduction in total risk by incurring a slight increase, merely 0.6%, in the travel cost to shipment 1. Not only the network under design (I) is an interesting solution for the regulator, it also involves small changes from the carriers' ideal network. Therefore, it can serve as a good starting point for the negotiation between the two parties, in our case study. The next alternative solution pertains to design (II) with the regulator's ability to close a section increased to 3 hours. Compared with design (I), this allows for imposing an additional curfew to the network, i.e., on section 1 during $[12pm, 3pm]$, only altering the shipment 2's route choice as shown in Figure D.2(c). In particular, shipment 2 switches to its 4^{th} best alternative by taking a longer route whose travel cost is 9.5% larger than the minimum-cost alternative. Nevertheless, this policy results in significant reductions in the hazmat transport risk- a 33% reduction over the unregulated scenario.

An interesting observation regarding design (III) is that closing 4 sections as in Figure D.2(d) does not make a remarkable improvement in the resulting transport risk compared with design (II). In fact, only a 0.25% further reduction can be achieved in total risk, which instead requires shipment 1 to incur an additional 29% increase in its travel cost due to a 2-hour delay in the departure time. Thus, one can conclude that design (II) dominates design (III) because both impose almost identical total risk while the latter involves higher average travel cost. Finally, another compromise solution is generated by the regulatory change (IV) leading to an over 44% decrease in the total risk. For such a reduction, however, the carriers will experience a 23% increase in the average travel cost which is mainly incurred by shipment 2 as it deviates to its 30^{th} best alternative.

The results of our case study problem further underscore the capability of the TD-HNDP framework in determining alternative solutions to the network design problem each corresponding to a certain trade-off between the transport risk and cost. This information does help the regulator in the decision-making process to account for the economic viability of the produced design policy, thereby

Table D.2: The unregulated scenario and the two alternative designs under the static network

Scenario	Total [a] Risk	Avg. Travel Cost \$ [c]	Shipment 1 Risk [b]	Shipment 1 Travel Cost \$ [c]	Shipment 1 Travel Time *hr.*	Shipment 2 Risk [b]	Shipment 2 Travel Cost \$ [c]	Shipment 2 Travel Time *hr.*
Unregulated	413.66	135.04	152.84	148.60	2.97	260.82	121.49	2.02
(A) 2 sections	273.13	317.21	149.83	469.32	9.39	123.30	165.10	2.75
(B) 4 sections	217.45	635.80	159.83	499.32	9.99	57.62	772.28	12.87

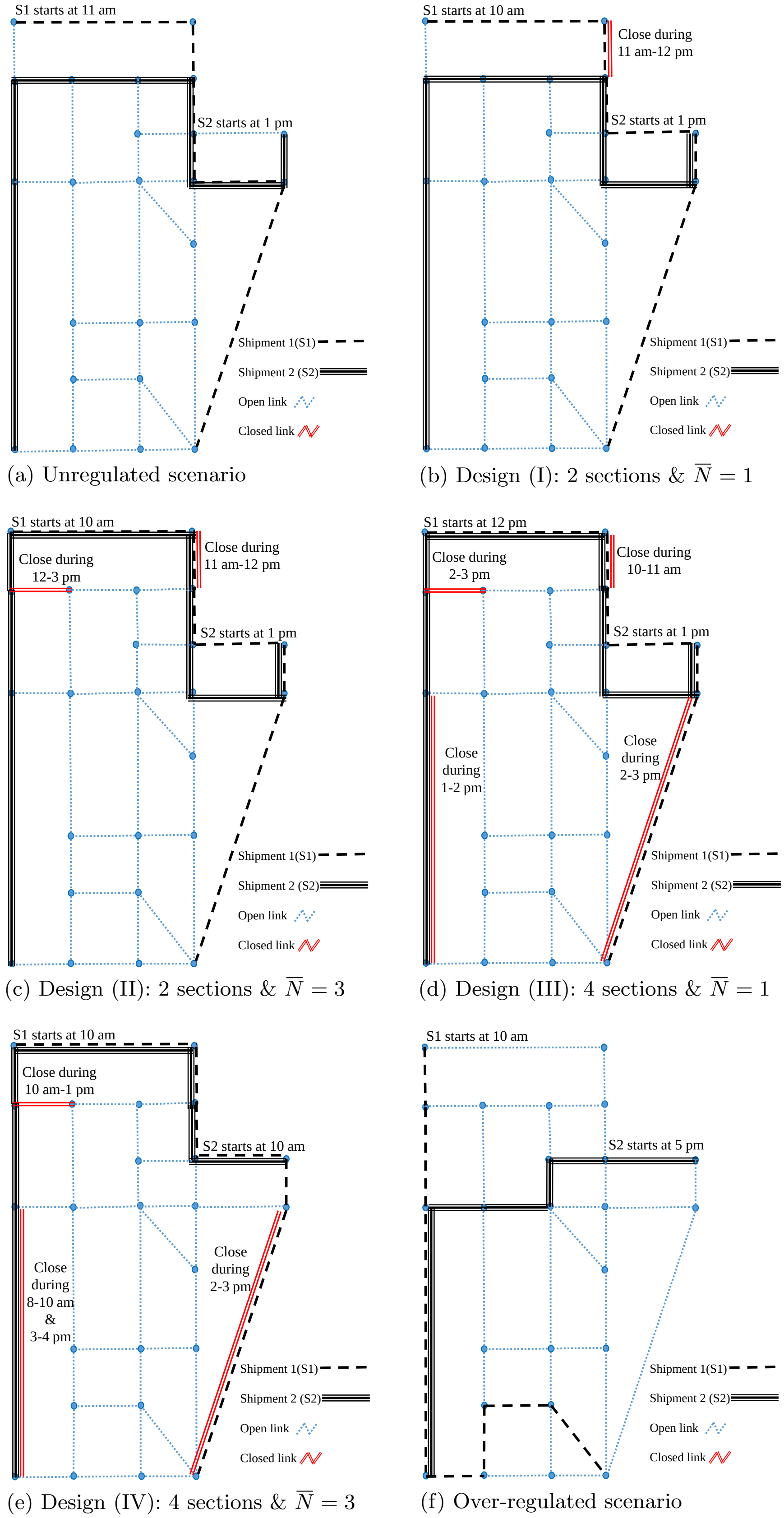
[a] Indicates the accumulated risk induced to the network, in *truck-people* unit and (10^{-4}) scale.
[b] Refers to the risk value induced to the network by the route-time decisions of the corresponding carriers, in *truck-people* unit and (10^{-4}) scale.
[c] Cost is calculated as $VoT \times$ *travel time*, since no penalty cost associated with delayed departure/ arrival can be realized in the static case.

ensuring the carriers' participation in the implementation phase. Note that the carriers' preferences during the policy design process might be specified by their maximum allowable percentage detour from the minimum-cost alternative (Verter and Kara, 2008). In our case study, assuming that both shipments are satisfied, for example, with risk-minimizing alternatives as long as they are at most 10% more expensive than their preferred alternatives, among the non-dominated schemes (I), (II) and (IV), the regulator can be convinced that design (II) is indeed very interesting for implementation because it produces acceptable solutions for the carries which are yet very attractive to the regulator.

D.3 The Results of the Static Network Design

To further investigate the benefits of the time-dependent network design framework, for our case study, we also analyze the network design problem under the assumption that the link attributes have no time variation. For this purpose, using the average attribute values, we solve the static version of the TD-HNDP by eliminating the time factor, with $\overline{N} = 1$ and $\epsilon = 10^{-5}$. In Table D.2, we summarize the characteristics of two network designs with: (A) *2 sections* and (B) *4 sections*, as well as the unregulated scenario. The road network available to hazmat shipments as a result of these policies and the carriers' routing decisions are illustrated in Figure D.3.

Our results from Table D.2 indicate that the exposure to hazmat risk can be substantially reduced under either of the two static network design policies- 34% and 47% reduction in total risk is achieved over the unregulated scenario, under scenario (A) and scenario (B), respectively. However, one observes that the price tag of these risk-mitigation policies would be over 134% and 370% increase in the average cost of the carriers. In particular, under scenario (A), shipment 1 is directed to its 18^{th}-best route whose travel cost is at least 200% above the minimum. This also involves a 36% increase in the travel cost of shipment 2. Imposing two more section closures under scenario (B) results in further deviations of the carriers' from their preferred scenarios; the routes depicted in Figure D.3(c) correspond to the 20th- and 74th-best routes of shipment 1 and 2, respectively. Clearly, such policies would not be feasible to the users, and require considerable expenditures of



(a) Unregulated scenario

(b) Design (I): 2 sections & $\overline{N} = 1$

(c) Design (II): 2 sections & $\overline{N} = 3$

(d) Design (III): 4 sections & $\overline{N} = 1$

(e) Design (IV): 4 sections & $\overline{N} = 3$

(f) Over-regulated scenario

Figure D.2: Time-based closure of roads and route-departure time decisions of the carriers under different regulatory schemes the regulator for inspecting compliance.

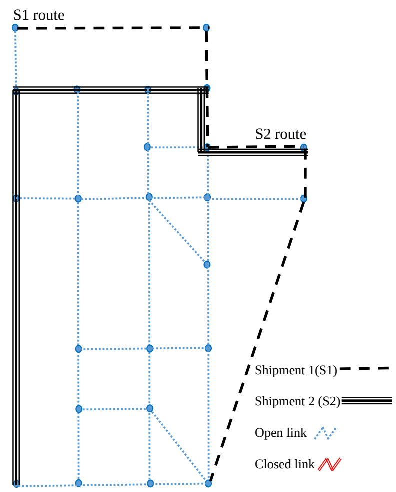
According to the results of Section D.2, in a time-varying framework, the regulator achieves significant reductions in transport risk without completely banning the use of roads. On the other hand, changes in trip departure times produce far less expensive solutions for the carriers compared with the static design polices. Note that unlike the permanent road closures of Figures D.3(b) and D.3(c) which completely prohibit the carriers from the use of certain links, the temporarily-closed road segments of Figures D.2(b)–D.2(e) can still be used by the carriers via delaying the departure times. This implies that a simultaneous analysis of route and departure time choices yields a much larger set of decision-making options, when compared with the static network. Further, the recognition of time-varying patterns of network parameters provides more accurate estimation of the risk and the cost than using the average attribute values (Nozick et al., 1997). Therefore, in a dynamic network, time-dependent road-closure puts forward an attractive policy to network regulators with more flexible solutions, and more acceptable choices to carriers, thereby creating a win-win scenario which is likely to lead to successful implementation. Although, solving the time-dependent version of the problem requires additional data collection efforts and more sophisticated analysis, the value seems compelling based on our observations.

E A Label Setting Algorithm for Solving Time-Dependent Elementary Shortest-Path Problems with Resource Constraints and Intermediate Stops

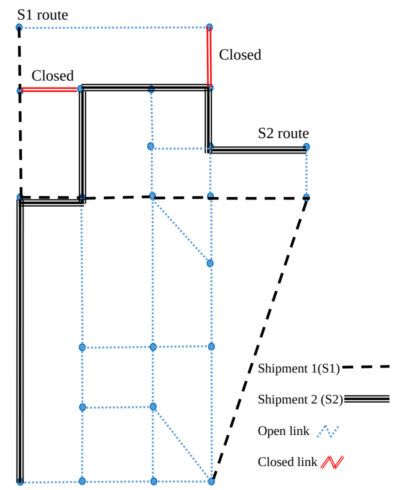
In generating the alternatives, our problem is a route-selection problem together with the determination of the departure times from the origin and the rest nodes subject to the constraints on waiting times and driving times. The single objective of this problem is to minimize either the trip travel time (including all driving and waiting times) or the risk. The former is when we wish to obtain the carriers' most proffered scenario, i.e., the minimum-cost alternatives. Whereas the latter obtains the minimum-risk alternatives constituting the most desirable scenario of the regulator.

All the generated alternatives should satisfy (i) the precedence relations, (ii) the restrictions on waiting times at a certain rest node, (iii) the constraint on the maximum permissible total duration of the trip, T_f, (iv) the constraint on the maximum uninterrupted driving time, D, and (v) the constraint on the maximum on-duty period, W. Consequently, the problem is indeed a time-dependent shortest path problem with resource constraints and intermediate stops (TD-SPPRC-IS) with time-varying arc impedances. We notice that unlike the total trip duration which is a non-renewable resource referring to the total time between emerging at the origin and arrival to the destination, the uninterrupted driving time and the on-duty period are two renewable resources, with the former being renewed after either short or long breaks, and the latter being renewed only after the long breaks.

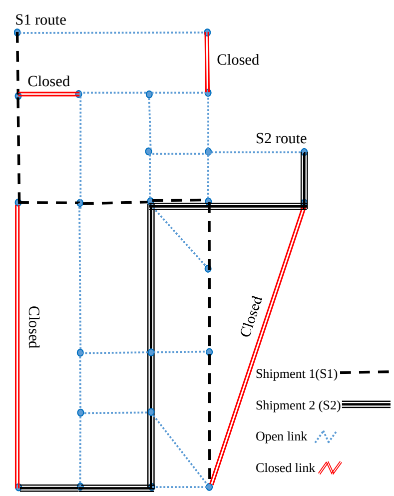
Based upon Feillet et al. (2004), we develop a dynamic programming approach that solves the TD-SPPRC-IS using a label setting algorithm for every departure time. We now turn to the



(a) Unregulated scenario



(b) scenario (A): 2 sections



(c) scenario (B): 4 sections

Figure D.3: Permanent closure of roads and routing decisions of the carriers under the static design policies of Table D.2

description of our algorithm. For an arbitrary shipment and a fixed departure time from the origin, the principle of our label setting algorithm is to associate with each partial possible alternative ending with node i a label indicating the consumptions of the resources. Taking into account the resource limitations, feasible extensions of every existing label are generated as the new labels. However, we do not create all the labels that pass the feasibility check due to there may be exponentially many of such labels. For this purpose, we eliminate the dominated labels with the help of dominance rules. The next two fundamental definitions constitute the building blocks of our label setting algorithm. In order to compare the labels more efficiently, we also add another resource. In particular, for a partial alternative ending at node i, the resource I_i identifies the index of the arrival time to node i. When one wishes to obtain the minimum-cost alternatives of the carriers, the following definitions are employed in the algorithm where the time-varying travel times are the arc impedances.

Definition 6 (Labels)**.** With each partial path p_{oi} from the origin node o to a node i, associate a label (state) $\{i; u_i, v_i, I_i\}$ with the first entry of the label being the last node of the partial path, the second being the last uninterrupted driving time on arrival at node i, the third being the duration of the current on-duty period up to node i, and I_i denoting the arrival time index to node i. The predecessor nodes and the waiting schedules of the corresponding label are represented by $\mathsf{Preds}\{i; u_i, v_i, I_i\}$ and $\mathsf{Wait}\{i; u_i, v_i, I_i\}$, respectively. The cost of this label is shown by $\mathsf{Time}\{i; u_i, v_i, I_i\}$ referring to the total travel time of the partial path from the origin to node i (including the driving times and waiting times at the origin and the rest areas) with waiting time at node i of zero.

With this definition of labels, the dominance relation in the comparison of two labels is stated as:

Definition 7 (Dominance Relation between Labels)**.** Let $\{i; u_1, v_1, I_1\}$ and $\{i; u_2, v_2, I_2\}$ be two distinct partial paths from the origin ending at the same node i with associated total travel times of $\mathsf{Time}\{i; u_1, v_1, I_1\}$ and $\mathsf{Time}\{i; u_2, v_2, I_2\}$, respectively. Label $\{i; u_1, v_1, I_1\}$ *dominates* label $\{i; u_2, v_2, I_2\}$ if $u_1 \leq u_2$, $v_1 \leq v_2$, $\mathsf{Time}\{i; u_1, v_1, I_1\} \leq \mathsf{Time}\{i; u_2, v_2, I_2\}$, and $I_1 = I_2$.

The above dominance rule is a derivation of the basic dominance rule proposed by Desrochers (1988) for the SPPRC in the static networks. In fact, to obtain the optimal solution of our problem, one needs to consider only the non-dominated labels. That is, if an extension of the second label is feasible, then the same extension will also be feasible for the first label with a lower cost. Thus the second label can be discarded. In a time-varying framework, an additional condition is required to assure the dominance of the first label, i.e., $I_1 = I_2$.

Although, in general, there may be exponentially many labels, a large subset of those cannot pass the feasibility check due to resource limitations. In addition, applying the dominance rule helps eliminate the dominated labels. Both of these observations result in considerable improvement in the computational effort. Inevitably, the proposed algorithm has exponential complexity. However, it solve the problem within 0.17 seconds (on average) for every departure time according to our numerical tests.

One can notice that a similar algorithm with a slight modification in the definition of the labels and the dominance rule is readily implementable to obtain the minimum-risk alternatives, i.e., the ideal scenario of the regulator. To this end, a label needs to account for the total travel time of an alternative as an additional constrained resource, whereas the cost of a label is the risk with the time-varying risk parameters constituting the arc impedances.

F The Label Setting Algorithm for Solving PSP

When applying the column-generation-based heuristic for the TD-HNDP-II, for solving the PSPs, one might suggest using a dynamic programming algorithm similar to Erkut and Alp (2007b, DP-IV) which can find the optimal minimum reduced cost alternative in the presence of complex restrictions on waiting and driving times. The generated alternative, however, is not guaranteed to be elementary, and extension of this approach for eliminating cycles is shown to further increase the computational complexity. In contrast, our experiments demonstrate that the label setting algorithm of Appendix E not only can account for the complex restrictions on waiting and driving times effectively, but also can be readily accommodated to assure obtaining elementary alternatives. Furthermore, the algorithm enables us to generate a set of good alternatives with *negative* reduced costs in every iteration of the column generation approach rather than the alternative with the most negative reduced cost, which is indeed beneficial for a fast evolving algorithm. The next two definitions are necessary for implementation of the label setting algorithm for solving PSP.

Definition 8 (Labels)**.** With each partial path p_{oi} from the origin node o to a node i, associate a label (state) $\{i; u_i, v_i, T_i, I_i\}$ with the first entry of the label being the last node of the partial path, the second being the last uninterrupted driving time on arrival at node i, the third being the duration of the current on-duty period up to node i, the forth referring to the total travel time of the partial path from the origin to node i (including the driving times and waiting times at the origin and the rest areas) with waiting time at node i of zero, and I_i denoting the arrival time index to node i. The predecessor nodes and the waiting schedules of the corresponding label are represented by $\mathsf{Preds}\{i; u_i, v_i, T_i, I_i\}$ and $\mathsf{Wait}\{i; u_i, v_i, T_i, I_i\}$, respectively. The cost of this label is shown by $\mathsf{RC}\{i; u_i, v_i, T_i, I_i\}$ referring to the reduced cost of the partial path from the origin to node i.

The reduced cost of a label is calculated by accumulating the time-varying modified risk of the arcs on the partial path. To generate feasible extensions of an existing label, among the unprocessed labels, we choose the label with the minimum total travel time, T_i. The *Preds* array of each partial path prevents visiting to nodes more than once, and thereby eliminates cycles. The dominated labels are discarded using the dominance relation stated as:

Definition 9 (Dominance Relation between Labels)**.** Let $\{i; u_1, v_1, T_1, I_1\}$ and $\{i; u_2, v_2, T_2, I_2\}$ be two distinct partial paths from the origin ending at the same node i with associated reduced costs of $\mathsf{RC}\{i; u_1, v_1, T_1, I_1\}$ and $\mathsf{RC}\{i; u_2, v_2, T_2, I_2\}$, respectively. Also let $\mathsf{Preds}\{i; u_1, v_1, T_1, I_1\}$ and $\mathsf{Preds}\{i; u_2, v_2, T_2, I_2\}$ denote the set of visited nodes of the two partial paths. Label $\{i; u_1, v_1, T_1, I_1\}$

dominates label $\{i; u_2, v_2, T_2, I_2\}$ if $u_1 \leq u_2$, $v_1 \leq v_2$, $T_1 \leq T_2$, $\mathsf{RC}\{i; u_1, v_1, T_1, I_1\} \leq \mathsf{RC}\{i; u_2, v_2, T_2, I_2\}$, $\mathsf{Preds}\{i; u_1, v_1, T_1, I_1\} \subset \mathsf{Preds}\{i; u_2, v_2, T_2, I_2\}$, and $I_1 = I_2$.

The above dominance rule is the adaptation of the modified dominance rule, which is prescribed for the ESPPRC in the static networks (see e.g., Chabrier, 2006; Nemani et al., 2010), to the time-varying networks guaranteeing optimality of the PSP algorithm. However, since obtaining the best alternative is not necessary during column generation, in our experiments, we relax the last two conditions to allow more labels being discarded, and thereby reduce the computational effort considerably.

G Benefits of Allowing Stops En Route

Based on the Buffalo network described in Section 5, we illustrate the benefits of allowing stops en route. We develop our discussion based upon two different scenarios:

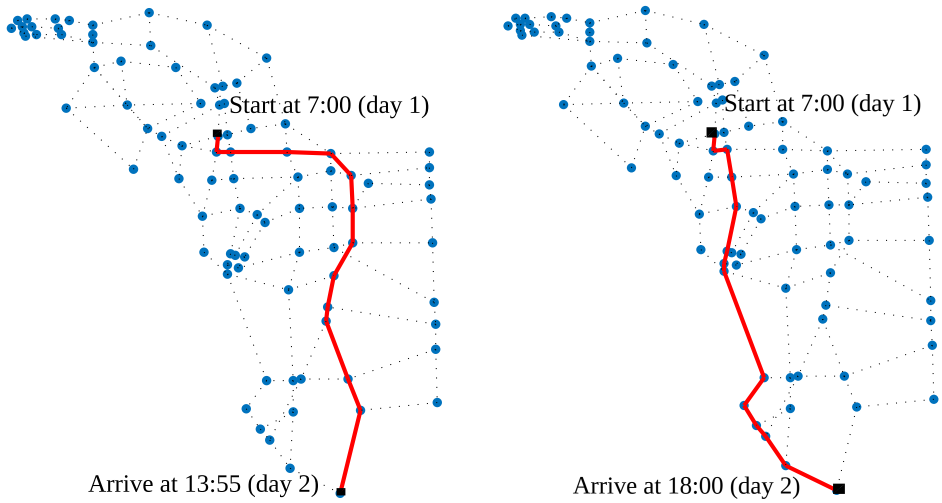
– *no-stopping scenario*: where waiting is allowed only at the origin node, but it is disallowed everywhere else along the route. In fact, the alternatives generated under this scenario are identical to the route-time alternatives defined as Definition 2 which are considered as carriers' decisions when designing a hazmat network by TD-HNDP.

– *stopping scenario*: where waiting is allowed at the origin node as well as at a restricted set of intermediate nodes along the route. This scenario constitutes alternatives of Definition 5 which are proposed to be employed in the TD-HNDP-II.

Under each scenario, we obtain the minimum-cost and the minimum-risk alternatives, i.e., the carriers' and the regulator's desired solutions, respectively. In particular, for the first scenario, we use the TDSPP algorithm of Ziliaskopoulos and Mahmassani (1993), with time and risk constituting the time-varying arc costs, respectively. For the second, however, we apply the label setting algorithm described in Section E with the objective of the problem being either time or risk.

Figure G.1 displays the routing and scheduling of the hazmat shipment found under no-stopping scenario with G.1(a) corresponding to the ideal solution of the carriers and G.1(b) being the preferred solution of the regulator. We summarize the characteristics of these two solutions in Table G.1.

As one can note from Table G.1, when waiting is not allowed along the route, the regulator can achieve 13% reduction in risk over the minimum-cost solution, by directing the carriers to the minimum-risk alternative. However, Figures G.1(a) and G.1(b) demonstrate that, to avoid the high-exposure areas, the minimum-risk alternative forces the carriers to take a path that is 18% more expensive than their preferred alternative. Furthermore, we observe that, despite the driving times of both of the alternatives prescribed by this scenario are within the permissible total trip duration, i.e., < 40 hrs., they require more than 30 hours of uninterrupted driving. Therefore, not only such alternatives are undesirable for hazmat transportation due to long trip duration, but also they are less likely to be undertaken by the carriers in reality due to workload constraints.



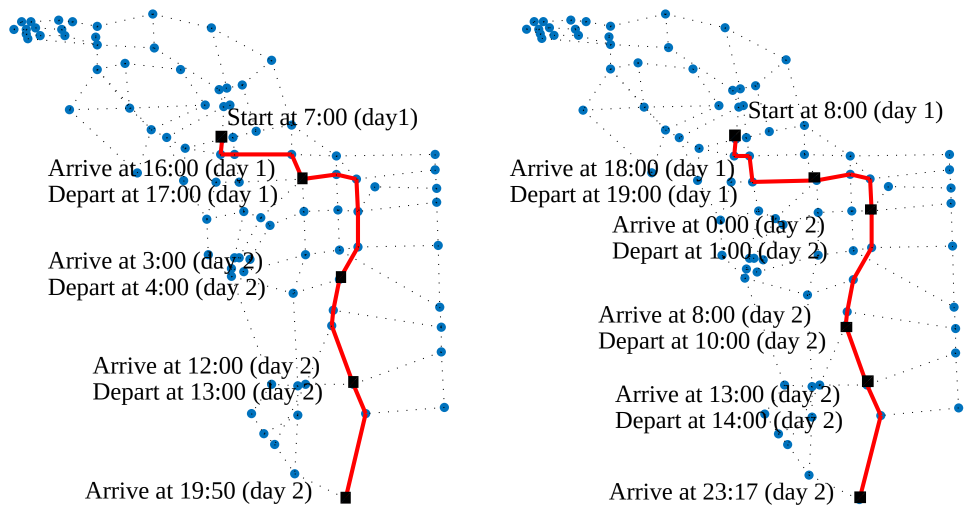
(a) The cost-minimizing alternative (b) The risk-minimizing alternative

Figure G.1: Routing and scheduling alternatives under the no-stopping scenario

Table G.1: Characteristics of the routing and scheduling alternatives generated under no-stopping scenario

		No-Stopping Scenario Alternatives	
Criterion	Unit	Minimum-Cost	Minimum-Risk
Risk	truck-people	6.05	5.25
Cost	$	44.87	52.99
Total trip time [a]	hour	30.94	34.99
Driving time	hour	30.94	34.99
Waiting time [b]	hour	0	0

[a] Refers to the total duration of the trip including the driving time and waiting time at the origin,
[b] Implies to duration of waiting at the origin node.



(a) The cost-minimizing alternative

(b) The risk-minimizing alternative

Figure G.2: Routing and scheduling alternatives under the stopping scenario, with $D = 10$ hrs., $L_{sb} = 0$, $U_{sb} = 3$ for maximum trip duration of 40 hours. The stops are indicated by black squares. Also note that the arrivals and departures are rounded to the nearest hour index for computation purposes.

We now analyze the results of the stopping scenario. The graphical representation of the proposed cost- and risk-minimizing routing-scheduling decisions that allow stops along the way are depicted in Figures G.2(a) and G.2(b), respectively. Table G.2 summarizes the implications of these alternatives.

We first compare the minimum-cost alternative of the stopping scenario with its counterpart in the no-stopping scenario. As expected, the minimum-cost alternative of the stopping scenario is comprised of an almost similar route (see Figures G.1(a) and G.2(a)), and the lowest possible amount of waitings (1 hour) being injected along the way to comply with the regulations. Inevitably, 3 hours of waiting would be necessary to complete the trip which results in the associated minimum-cost being 25% higher than its counterpart in the no-stopping scenario. Nonetheless, not only this alternative is more practical, but also an interesting observation is that, when stops are allowed, the minimum-cost alternative induces lower risk to the network- an 8% reduction over the no-wait solution. This is due to, when stopping is allowed, the vehicle can take a break at a rest node to avoid congestion in order to minimize the travel time (cost) which also results in less exposure. However, if stopping is not allowed, the vehicle would either drive during the rush hour or take a detour which can increase the exposure.

Comparing the alternatives depicted in Figures G.2(a) and G.2(b), we notice that, under the stopping scenario, the minimum-risk alternative devises an only slightly different route than that of the minimum-cost alternative with the driving times being almost identical. Therefore, the 12% increase in the cost over the minimum-cost alternative is only due to waitings. In fact, constrained by the maximum trip duration of 40 hours, the minimum-risk alternative benefits from the stops

Table G.2: Characteristics of the routing and scheduling alternatives generated under stopping scenario for maximum trip duration of 40 hours

Criterion	Unit	Stopping Scenario Alternatives	
		Minimum-Cost	Minimum-Risk
Risk	truck-people	5.56	5.42
Cost	$	56.29	63.23
Total trip time [a]	hour	36.64	39.61
Driving time	hour	33.64	33.61
Waiting time [b]	hour	3.00	6.00

[a] Refers to total duration of the trip including the driving time, as well as the waiting times at the origin and rest areas,
[b] Implies to total duration of waiting at the origin node and the rest areas.

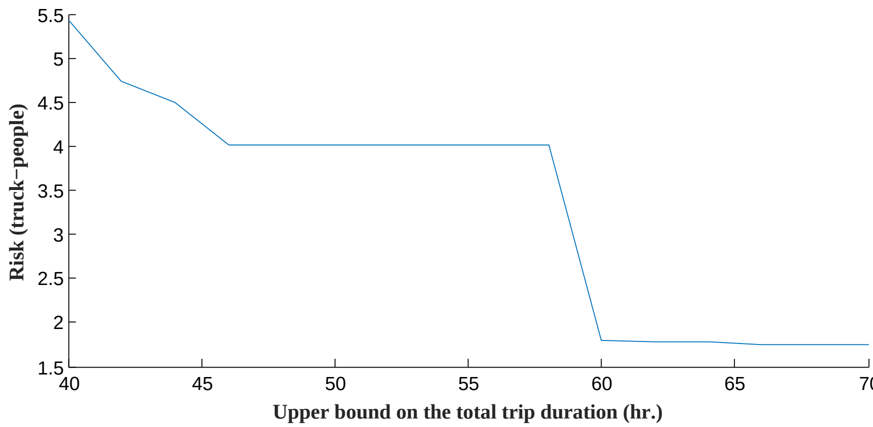
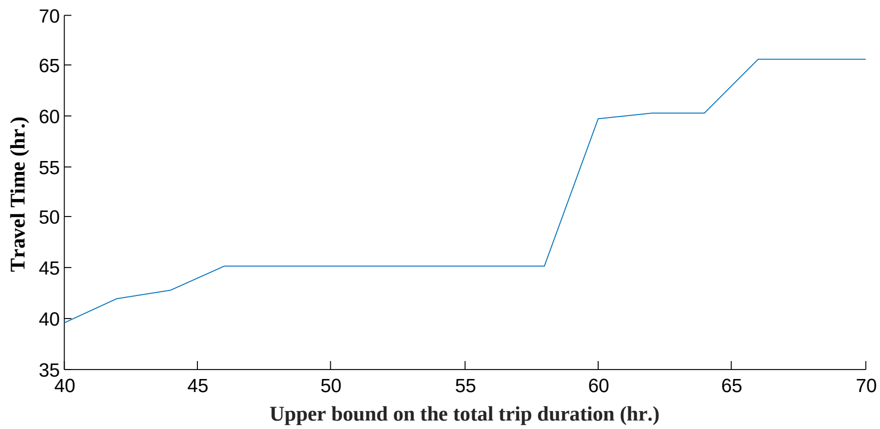


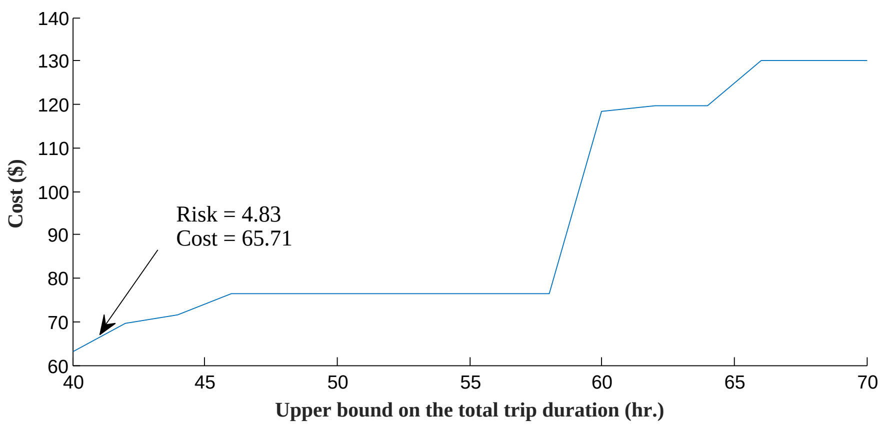
Figure G.3: Impact of the maximum allowable trip duration on the risk reduction

to avoid high-exposure travel intervals without taking a complete detour which is the case in the no-stopping scenario. In particular, an 1-hour delay at the origin and 5 hours of rest/ wait stops at 4 intermediate nodes result in 2.5% reduction in risk. As more time is allowed for the trip, the risk-minimizing route and schedule decision can take more advantage of the time-varying network attributes thorough incorporation of stops. To show this, we regenerate the minimum-risk alternative subject to the constraint that the trip duration is no more than t hours, for all $40 \leq t \leq 70$. Figure G.3 displays the impact of increasing the allowable trip duration on the risk reduction. Note that the risk gradually decreases as the trip duration increases to 46 hours. The reduction is steady between 46 and 58 hours, dramatic between 58 and 60 hours, and very little after 60 hours. Increasing the maximum trip duration, however, is more likely involved with increasing the total travel time and consequently the travel cost due to the excess waiting times along the way. Figures G.4(a) and G.4(b) illustrate this behavior.

To allow comparison with the results of the no-stopping scenario, we obtained the minimum-risk alternative that achieves 13% reduction in risk over the minimum-cost alternative. This is when



(a) Total travel time versus trip duration



(b) Travel cost versus trip duration

Figure G.4: Impact of the maximum allowable trip duration on the total travel time and cost of the minimum-risk alternative the upper bound on the trip duration is set to 42 hours (see Figure G.4(b)). For such a reduction, however, the carriers would experience 16.7% increase in their travel cost, which is a less expensive solution than its counterpart in the no-stopping scenario.

The overall conclusion of our discussion is that the en route stops can take full advantage of the time-varying nature of the data, and are likely to improve the routing and scheduling decisions in terms of the cost and the associated transport risk. Not only can waiting at nodes be more effective than the no-wait case in reducing hazmat transport risk which is beneficial from the regulator's perspective, it can also be associated with more cost-efficient solutions for the carriers. Furthermore, allowing for stopping at intermediate nodes is a more realistic assumption as, in practical situations, they are mandated by the authorities for completing long trips. Therefore, we suggest incorporating stops in the routing and scheduling decisions, i.e., alternatives, of the carriers when designing a time-dependent hazmat network in TD-HNDP-II. We expect such modification to generate time-dependent closure policies which are more desirable for risk-reduction yet less expensive to the carriers.