Exact Robust Solutions for the Combined Facility Location and Network Design Problem in Hazardous Materials Transportation

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Abstract

We consider a leader-follower game in the form of a bi-level optimization problem that simultaneously optimizes facility locations and network design in hazardous materials transportation. In the upper level, the leader intends to reduce the facility setup cost and the hazmat exposure risk, by choosing facility locations and road segments to close for hazmat transportation. When making such decisions, the leader anticipates the response of the followers who want to minimize the transportation costs. Considering uncertainty in the hazmat exposure and the hazmat transport demand, we consider a robust optimization approach with multiplicative uncertain parameters and polyhedral uncertainty sets. The resulting problem has a min-max problem in the upper level and a shortest-path problem in the lower level. We devise an exact algorithm that combines a cutting plane algorithm with Benders decomposition.

Keywords: bi-level optimization; facility location; network design; cutting plane; Benders decomposition; hazardous materials

1 Introduction

Hazardous materials (hazmat) are "solids, liquids, or gases that are harmful to people, property, and the environment" (United Nations, 2009). A large amount of hazmat are generated in industrial production and transported over various transportation modes. Trucks are the most popular mode of transporting hazmat (Erkut et al., 2007). For example, in the U.S., more than 2.4 billion tons of hazmat were transported by trucks in 2012 (U.S. Department of Transportation, 2015). Accidents

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involving hazmat can create catastrophic consequences; hence the road system is facing pressure on the constantly increasing amount of hazmat shipments. Managing risk in hazmat transportation is important in any industrial society.

In most cases, the hazmat producers are responsible to carry hazmat to an appropriate processing facility. The hazmat carriers make their choices about the transportation route, usually, aiming to minimize the shipment cost. The local route decision of hazmat carriers is beyond the control of the government, who considers the impact of hazmat transportation from a global perspective of managing the entire road network and other infrastructure systems. The government wants to minimize the total shipment exposure risk and total facility construction costs. To achieve this goal, the government may consider road-ban policies to specify the available and unavailable roads for hazmat shipments. Such policies prohibit hazmat carriers from choosing a route with small transportation costs but with great hazmat exposure risk. The problem to determine such road-ban policies is called a hazmat network design problem in the literature.

In this paper, we consider a *combined* hazmat facility locations and network design problem. We assume that origin points where hazardous materials are produced are known, but destination points (disposal facility location) are not. Instead, hazmat carriers are assumed to choose the nearest facility if multiple facilities are available within the network; therefore, the route decision of hazmat carriers is dependent on the location decision of the government. When the government determines the locations of hazmat processing facilities, we assume that the government also considers a road-ban policy to design the hazmat network, upon which the route decision of hazmat carriers also depends. This structure of hierarchical decision-making has been considered in a bi-level optimization framework in the literature (Kara and Verter, 2004; Erkut and Alp, 2007; Gzara, 2013; Berglund and Kwon, 2014; Marcotte et al., 2009; Sun et al., 2015). We will present our problem as a bi-level optimization problem as well.

We consider uncertain hazmat transportation demands and uncertain hazmat accident risks. By assuming data for the demands and risks are available as intervals, we consider the worst-case scenario using a robust optimization approach. We will consider polyhedral uncertainty sets as considered in Bertsimas and Sim (2003). In our problem, the two uncertain parameters form a product in the objective function, for which we adopt the approach of Kwon et al. (2013).

Our work is closely related to Berglund and Kwon (2014) and Gzara (2013). Berglund and Kwon (2014) have considered a robust hazmat facility location problem. Our modeling approach for the robust combined facility location and network design problem extends the work of Berglund and Kwon (2014). The computational method proposed by Berglund and Kwon (2014), however, is a genetic algorithm, which does not produce an exact optimal solution in general. In this paper, for the combined problem, we devise an exact algorithm by adopting the cutting plane algorithm of Gzara (2013) and combining with Benders decomposition.

Gzara (2013) has devised a cutting plane algorithm for solving the bi-level hazmat network design problem. The model of Gzara (2013), however, only considered a network design decision without considering data uncertainty. Our problem is a robust optimization problem that considers the facility location decision and the network design decision jointly. As we adopt the cutting plane algorithm of Gzara (2013) to the robust combined problem, we have revised the cut generation method for the joint decision. We also simplify the inequalities in the cuts and eliminate the need for additional binary variables. In our problem, the master problem is significantly harder to solve, mainly due to the robustness consideration; we devise a Benders decomposition (Benders, 1962) approach for solving the master problem. While a Benders decomposition approach has been used to solve a single-level reformulation of the deterministic hazmat network design problem (Fontaine and Minner, 2018), we use Benders decomposition to solve the robust master problem involving uncertainty within the cutting plane algorithm framework for the joint decision of facility location and network design.

The contributions of this paper are summarized as follows. We consider a combined facility location and network design problem for hazmat transportation. By assuming data uncertainty, we formulate a robust optimization problem as a bi-level mixed-integer optimization problem, where the upper-level problem has a min-max structure. We propose a cutting plane algorithm incorporated with Benders decomposition to solve the robust combined problem.

The remainder of this paper is as follows. In Section 2, more related works are summarized and the relevance to our work is discussed. In Section 3, a bi-level location-network design mathematical optimization model is formulated. In Section 4, we present a cutting plane algorithm, combined with Benders decomposition, to solve the optimization problem. In Section 5, we provide a single-level reformulation of the bi-level robust problem. Results from numerical experiments are discussed in Section 6. Finally, conclusions and future researches are provided in Section 7.

2 Literature Review

In this section, we review the literature in the four categories: hazmat facility location, hazmat network design, combined facility and network design in non-hazmat context, and robust optimization approaches in hazmat transportation.

2.1 Hazmat Facility Location Problems

There are a variety of methods for facility location problem in hazmat transportation. The related studies assume that facility locations are not given and need to solve a routing problem. Carotenuto et al. (2007) propose two greedy algorithms to select the path which minimizes the total risk. Xie et al. (2012) study multi-objective hazmat model that optimizes facility locations and routes in the long-distance transportation and solve the mixed integer linear program by CPLEX. Jarboui et al. (2013) propose various neighborhood search (VNS) heuristics for solving location-routing problem. Samanlioglu (2013) studies a location-routing problem and propose a lexicographic weighted Tchebycheff formulation to minimize multi-objectives of total cost, transportation risk, and site risk. Ardjmand et al. (2015) apply a novel genetic algorithm for location-routing problem in facilities and disposal sites. Romero et al. (2016) analyze location-routing decisions considering equity based on Gini coefficient and propose a method that combines Lagrangian relaxation with column generation. Rabbani et al. (2018) emphasize on hazmat formulation restriction, i.e., incompatibility between different kinds of waste with multi-objectives of minimizing total cost, transportation risk, and site risk. They use Nondominated Sorting Genetic Algorithm (NSGA-II) and Multi-Objective Particle Swarm Optimization (MOPSO) to solve the problem. For earlier works, see Berglund and Kwon (2014) and references therein.

2.2 Hazmat Network Design Problems

There are also some research papers related to network-design problem. The routing is also considered when the locations of origin-destination pairs are given. Verter and Kara (2008) provide a path-based formulation for network design hazmat shipment problem and compromise between exposure risk and economic viability. Garrido (2008) and Marcotte et al. (2009) study a network-design problem where origin-destination pairs are given and aim to minimize exposure risk. They design the network by road pricing method, and Wang et al. (2012) improve the method and propose a dual-toll pricing policy. Bianco et al. (2009) provide a linear bi-level programming formulation for the hazmat transportation network design that considers minimizing total risk and risk equity. They propose a heuristic algorithm to find a stable solution. Gzara (2013) proposes a family of valid cuts and incorporates with an exact cutting plane algorithm for solving a bi-level network flow model. Bianco et al. (2015) study a novel toll setting policy and formulate a mathematical programming with equilibrium constraints where the government aims to minimize total risk and carriers intend to minimize travel cost. Taslimi et al. (2017) propose a bi-level network design model with the aim to minimize the maximum zone total risk and propose a greedy heuristic approach for large-size problems. Esfandeh et al. (2017) formulate the time-dependent network design problem based on altering carriers' departure times and route choices and extend the model that can consider consecutive time-based road closure policies and allow carriers to stop at the intermediate nodes.

2.3 Combined Facility Location and Network Design Problem in Non-hazmat Context

To the best of our knowledge, there are few papers related to combined facility location and network design problem in hazmat transportation; we review some relevant papers in non-hazmat context. The main difference between hazmat and non-hazmat problems is that hazmat problems usually need to be in the bi-level form with hierarchical decision-making.

Melkote and Daskin (2001b) investigate a generalized model that optimizes facility location and transportation network. Then they extend the model when facilities have capacity constraint and present several classes of valid inequalities to strengthen its LP relaxation (Melkote and Daskin, 2001a). Ravi and Sinha (2006) propose an approximation algorithm for combined facility location and network design problem with minimizing facilities opening costs and transportation costs. Gelareh and Pisinger (2011) formulate a mixed integer linear programming for deep-sea liner service providers' locations and network design and propose a primal decomposition method. Contreras et al. (2012) present two mixed integer programming formulations which generalize the classical p-center problem in order to minimize the maximum customer-facility travel time. Ghaderi and Jabalameli (2013) present a model for the budget-constrained facility location–network design healthcare problem with minimizing multi-objectives of total travel costs and operating costs for facilities and network arcs. And a greedy heuristic is proposed based on simulated annealing and cutting plane method. Rahmaniani and Ghaderi (2013) propose a fix-and-optimize heuristic to solve bi-objective combined facility location and network design problem with capacitated arcs. Ghaderi (2015) studies a facility location-network design problem over several different time periods in order to minimize the maximum travel time between each pair of origin-destination and proposes an improved Variable Neighborhood Search.

2.4 Robust Optimization Approaches in Hazmat Transportation

In hazmat transportation problems, considering data uncertainty is necessary (Kwon et al., 2013). Stochastic programming methods are, however, less effective, because historical data are often insufficient to construct probability distributions for the risk exposure. When probability distributions of uncertain parameters are unknown, robust optimization is a useful technique (Bertsimas and Sim, 2003). Killmer et al. (2001) study a noxious facility location problem involving uncertainty by a robust optimization method. Sharma et al. (2009) formulate and solve the multi-objective robust network design problem with uncertain demand. Berglund and Kwon (2014) consider a robust facility location and routing problem for hazardous materials management with the objective of minimizing the total cost and also analyze the impact of uncertainty in the demand and exposure risk. Xin et al. (2015) use robust optimization method to formulate a bi-level model under risk values uncertainty for designing hazmat transportation network. Sun et al. (2015) study a robust hazmat network design problem considering risk uncertainty and devise a heuristic method with Largrangian relaxation. Sun et al. (2017) consider behavioral uncertainty from hazmat carriers and formulate a robust optimization problem, for which a cutting plane algorithm is devised.

3 The Robust Combined Facility Location-Network Design Problem

We consider a graph $G(\mathcal{N}, \mathcal{A})$ where $\mathcal{N}$ is the set of nodes and $\mathcal{A}$ is the set of directed arcs. We assume that the sources of hazmat are at the known subset of nodes in the network, but the destinations (disposal facility) are not. We let $\mathcal{S}$ denote the set of hazmat shipments and $o(s)$ denote the origin node of shipment $s \in \mathcal{S}$. We want to determine the proper number and locations for constructing facilities from a set of the candidate facility sites. Note that we assume disposal facilities do not generate hazmat; i.e., $\bigcup_{s \in \mathcal{S}} o(s) \cap \mathcal{M} = \emptyset$, where $\mathcal{M}$ denotes the set of candidate facility locations. At the same time, we will consider a road-ban policy by network designer. The upper level objective function is to minimize a linear combination of fixed facility cost and the worst-case exposure risk. The lower level objective function is to minimize the transportation cost for hazmat carriers. We assume that the hazmat carriers choose the least cost route to the nearest hazmat facility.

For shipment s, the expected number of trucks required is N^s. We let the anticipated risk induced by each truck for shipment s on arc (i,j) is R^s_{ij}. While the population exposure is a popular choice for the risk measure R^s_{ij}, one may use other metrics such as the accident probability and the environmental impact. We require the risk measure R^s_{ij} to hold linearity and additivity properties that ensures risk being measurable as the linear combination of metrics.

While one can estimate N^s and R^s_{ij} based on a survey and the national averages, the values of these two critical parameters are hardly known exactly (Berglund and Kwon, 2014). To address such data uncertainty, we employ a robust optimization approach. Following Berglund and Kwon (2014), we assume that the demand is given as an interval $[N^s, N^s + K^s]$ and the risk as $[R^s_{ij}, R^s_{ij} + Q^s_{ij}]$.

We denote the routing variable of hazmat carriers by $\mathbf{x}$, where $x^s_{ij} = 1$ if arc (i,j) is chosen for shipment s and $x^s_{ij} = 0$ otherwise. The worst-case total risk can be modeled as follows:

$$\max_{\mathbf{u}\in\mathbf{U},\mathbf{v}\in\mathbf{V}} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} (N^s + K^s u^s)(R^s_{ij} + Q^s_{ij} v_{ij}) x^s_{ij}$$

where the uncertainty sets $\mathbf{U}$ and $\mathbf{V}$ are bounded. The uncertain variables $\mathbf{u}$ and $\mathbf{v}$ are constrained to stay within a specific range, and the total deviation from nominal values is limited by a budget of uncertainty. In particular, we define the uncertainty sets with the budget of uncertainty, Γ_u and Γ_v, as follows:

$$\mathbf{U} = \left\{ \mathbf{u} : \sum_{s\in S} u^s \le \Gamma_u, \quad 0 \le u^s \le 1 \right\}$$
$$\mathbf{V} = \left\{ \mathbf{v} : \sum_{(i,j)\in A} v_{ij} \le \Gamma_v, \quad 0 \le v_{ij} \le 1 \right\}.$$

Using the notation introduced in Table 1, we formulate the robust combined location-network design problem as the following bi-level optimization problem:

$$\underset{\mathbf{y},\mathbf{z}}{\text{minimize}} \quad \left[w_1 \sum_{i\in\mathcal{M}} F_i y_i + w_2 \max_{\mathbf{u}\in\mathbf{U},\mathbf{v}\in\mathbf{V}} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} (N^s + K^s u^s)(R^s_{ij} + Q^s_{ij} v_{ij}) x^s_{ij} \right] \tag{1}$$

$$\text{subject to} \quad y_i \in \{0,1\} \qquad \forall i \in \mathcal{M} \tag{2}$$

$$z_{ij} \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A} \tag{3}$$

where $\mathbf{x}$ solves

$$\underset{\mathbf{x}}{\text{minimize}} \quad \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} c_{ij} x^s_{ij} \tag{4}$$

$$\text{subject to} \quad \sum_{j:(i,j)\in\mathcal{A}} x^s_{ij} - \sum_{j:(j,i)\in\mathcal{A}} x^s_{ji} \begin{cases} = 1 & \text{if } i = o(s) \\ \ge -y_i & \text{if } i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{N}, s \in \mathcal{S} \tag{5}$$

Table 1: Mathematical Notation

Sets	
$\mathcal{N}$	the set of nodes
$\mathcal{A}$	the set of arcs
$\mathcal{S}$	the set of hazmat shipments
$\mathcal{M}$	the set of candidate facility locations
$\mathcal{K}$	the set of chosen facility locations
Parameters	
c_{ij}	the cost of transportation though arc $(i,j) \in \mathcal{A}$
R_{ij}^s	the measure of exposure risk of shipment $s \in \mathcal{S}$ through arc $(i,j) \in \mathcal{A}$
$o(s)$	the node where hazmat are generated for shipment $s \in \mathcal{S}, o(s) \cap \mathcal{M} = \emptyset$
F_i	the cost of constructing a hazmat processing facility at node $i \in \mathcal{M}$
N^s	the number of trucks required for shipment $s \in \mathcal{S}$
Γ_u	the budget of uncertainty in the number of trucks
Γ_v	the budget of uncertainty in exposure risk
K^s	the width of the uncertainty in the number of trucks required by shipment $s \in \mathcal{S}$
Q_{ij}^s	the width of the uncertainty in the exposure risk through arc $(i,j) \in \mathcal{A}$
Variables	
x_{ij}^s	1, if arc $(i,j) \in \mathcal{A}$ is chosen for shipment $s \in \mathcal{S}$; 0, otherwise.
y_i	1, if a facility is located at node $i \in \mathcal{N}$; 0, otherwise.
z_{ij}	1, if arc $(i,j) \in \mathcal{A}$ is available for shipments; 0, otherwise.
u^s	the uncertainty variable for the number of trucks required for shipment $s \in \mathcal{S}$.
v_{ij}	the uncertainty variable for the exposure risk through arc $(i,j) \in \mathcal{A}$.

$$x_{ij}^s \leq z_{ij} \qquad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \tag{6}$$

$$x_{ij}^s \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}. \tag{7}$$

Note that since facilities construction cost and exposure risk are not directly comparable, we will make a trade-off between these two parts of the objective function, i.e., set a dollar amount equal to a unit of exposure risk. If the decision maker is prone to avoid risk, he/she can set a higher dollar cost equal to a unit of risk, and vice versa. In the upper level objective function (1), w_1 and w_2 represent the weight for cost and risk. The first part is the total facility construction cost and the second part represents the worst-case risk. Without loss of generality, we assume $(w_1, w_2) = (1, 1)$ for the rest of this paper.

The lower level objective function (4) is to minimize the carriers' own shipment cost. Constraint (5) ensures that origin nodes must have net outflow of 1; when node i is selected as a facility ($y_i = 1$), node i can have net outflow of either -1 if node i is chosen as a destination or 0 otherwise; when node i is not selected as a facility ($y_i = 0$), node i is same as an intermediate node with zero net outflow; and all other intermediate nodes must have a zero balance. Constraint (6) means that selecting arc (i, j) is constrained by whether it is available ($z_{ij} = 1$) or not ($z_{ij} = 0$). Constraints (2), (3), and (7) represent that routing variable $\mathbf{x}$, location variable $\mathbf{y}$, and network design variable $\mathbf{z}$ are binary variables.

The bi-level optimization problem, where the upper-level problem is a min-max problem, can be formulated as a single-level optimization problem, shown in Section 5. The resulting single-level problem may be solved by off-the-shelf optimization solvers such as Gurobi and CPLEX, when the problem instance is small. For large problems, optimization solvers struggle with computational difficulty as shown in Section 6. There is also an issue with big-M in the single-level problem.

4 An Exact Solution Method

To solve the bi-level mixed integer program problem, we propose a cutting plane algorithm based on the cuts in Gzara (2013) and the idea of transforming location-network design problem into a pure network design problem from Melkote and Daskin (2001b). The nature of the cutting plane algorithm is to compare upper level objective (the Government's global goal) path and lower level objective (carrier's goal) path. When these two paths are same, an optimal solution is obtained. While the cutting plane algorithm can separate the lower-level problem as a subproblem from the upper-level master problem, the master problem is a computationally challenging problem, mainly due to the worst-case consideration in the upper-level objective. To tackle such difficulty, we use a Benders decomposition approach for solving the master problem. To distinguish master and subproblem from the cutting plane algorithm and Benders decomposition, we use C-Master/C-Sub and B-Master/B-Sub, respectively. We illustrate the entire computational framework in Figure 1. The dotted line represents the original flow in the cutting plane algorithm of Gzara (2013), which is replaced by Benders decomposition in this paper. Note that generated cuts in C-Master are carried

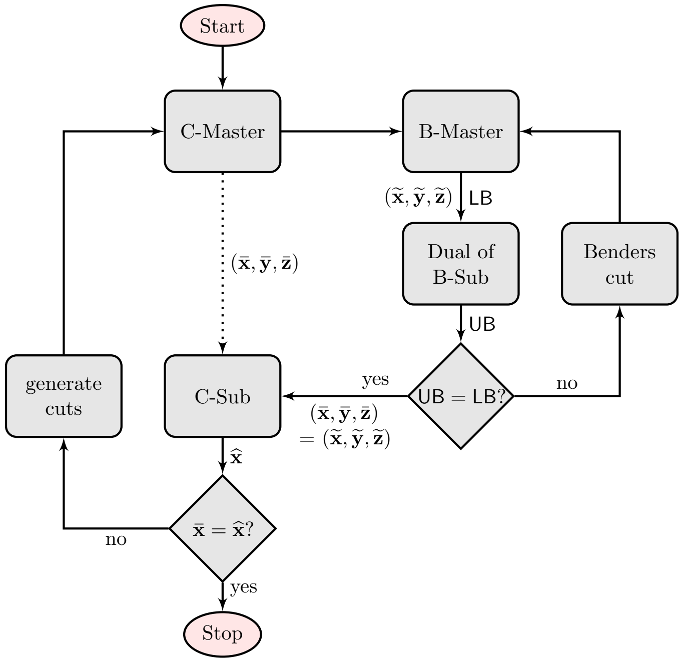


Figure 1: Flow Chart for the Cutting Plane Algorithm combined with Benders decomposition.

over to B-Master, while Benders cuts are not carried over to C-Master.

4.1 Cutting Plane Algorithm

The master problem obtains the minimization of facility construction cost and the total shipment risk. The valid cuts (Section 4.2) will be added to C-Master iteratively. By adding cuts, network design variables z_{ij} can be changed to ensure carriers not to choose a certain arc. C-Master is firstly formulated as follows:

$$
\begin{aligned}
\underset{\mathbf{x},\mathbf{y},\mathbf{z}}{\text{minimize}} \; & \left[\sum_{i \in \mathcal{M}} F_i y_i + \max_{\mathbf{u},\mathbf{v}} \sum_{(i,j)\in\mathcal{A}} \sum_{s \in \mathcal{S}} (N^s + K^s u^s)(R^s_{ij} + Q^s_{ij} v_{ij}) x^s_{ij} \right] \\
\text{subject to} \; & \sum_{j:(i,j)\in\mathcal{A}} x^s_{ij} - \sum_{j:(j,i)\in\mathcal{A}} x^s_{ji} \begin{cases} = 1 & \text{if } i = o(s) \\ \geq -y_i & \text{if } i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} && \forall i \in \mathcal{N}, s \in \mathcal{S} \\
& x^s_{ij} \leq z_{ij} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& x^s_{ij} \in \{0,1\} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& y_i \in \{0,1\} && \forall i \in \mathcal{M} \\
& z_{ij} \in \{0,1\} && \forall (i,j) \in \mathcal{A} \\
& \sum_{s \in S} u^s \leq \Gamma_u
\end{aligned}
$$

$$\sum_{(i,j)\in A} v_{ij} \leq \Gamma_v$$

$$0 \leq u^s \leq 1 \qquad \forall s \in \mathcal{S}$$

$$0 \leq v_{ij} \leq 1 \qquad \forall (i,j) \in \mathcal{A}$$

additional cuts (23) (Section 4.2) added

Note that the above problem is a robust optimization problem for combined facility location-network design decisions, with additional cuts generated from the lower-level sub problem. To reformulate this problem as a single-level problem, we use dualization and linearization techniques introduced in Kwon et al. (2013). The inner maximization part can be expanded as follows:

$$\begin{aligned}
&\max_{\mathbf{u}\in\mathbf{U},\mathbf{v}\in\mathbf{V}} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} (N^s + K^s u^s)(R^s_{ij} + Q^s_{ij} v_{ij}) x^s_{ij} \\
&= \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} N^s R^s_{ij} x^s_{ij} + \max_{\mathbf{u}\in\mathbf{U},\mathbf{v}\in\mathbf{V}} \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} (N^s Q^s_{ij} v_{ij} + K^s R^s_{ij} u^s + K^s Q^s_{ij} u^s v_{ij}) x^s_{ij}
\end{aligned}$$

For any give $\mathbf{x}$, the inner maximization problem is equivalent as follows:

$$\begin{aligned}
\underset{\mathbf{u},\mathbf{v}}{\text{maximize}} \quad & \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} (N^s Q^s_{ij} v_{ij} + K^s R^s_{ij} u^s + K^s Q^s_{ij} u^s v_{ij}) x^s_{ij} \\
\text{subject to} \quad & \sum_{s\in S} u^s \leq \Gamma_u, \quad 0 \leq u^s \leq 1 \\
& \sum_{(i,j)\in A} v_{ij} \leq \Gamma_v, \quad 0 \leq v_{ij} \leq 1
\end{aligned}$$

By letting w^s_{ij} represent the quadratic term $u^s v_{ij}$ for each $(i,j) \in \mathcal{A}, s \in \mathcal{S}$, the above model can be linearized as follows:

$$\begin{aligned}
\underset{\mathbf{u},\mathbf{v}}{\text{maximize}} \quad & \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} (N^s Q^s_{ij} v_{ij} + K^s R^s_{ij} u^s + K^s Q^s_{ij} w^s_{ij}) x^s_{ij} \\
\text{subject to} \quad & u^s \leq 1 && \forall s \in \mathcal{S} && (\rho^s) \\
& v_{ij} \leq 1 && \forall (i,j) \in \mathcal{A} && (\xi_{ij}) \\
& -u^s + w^s_{ij} \leq 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} && (\eta^s_{ij}) \\
& -v_{ij} + w^s_{ij} \leq 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} && (\pi^s_{ij}) \\
& \sum_{s\in S} u^s \leq \Gamma_u && && (\theta_u) \\
& \sum_{(i,j)\in A} v_{ij} \leq \Gamma_v && && (\theta_v) \\
& u^s \geq 0 && \forall s \in \mathcal{S} \\
& v_{ij} \geq 0 && \forall (i,j) \in \mathcal{A}
\end{aligned}$$

The dual variables ρ^s, ξ_{ij}, η_{ij}^s, π_{ij}^s, θ_u and θ_v are introduced. The dual problem of the above problem becomes:

$$
\begin{aligned}
\underset{\boldsymbol{\rho},\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{\pi},\theta_u,\theta_v}{\text{minimize}} \quad & \sum_{s\in S}\rho_s + \sum_{(i,j)\in\mathcal{A}}\xi_{ij} + \Gamma_u\theta_u + \Gamma_v\theta_v \\
\text{subject to} \quad & \rho^s - \sum_{(i,j)\in\mathcal{A}}\eta_{ij}^s + \theta_u \geq \sum_{(i,j)\in\mathcal{A}} K^s R_{ij}^s x_{ij}^s && \forall s\in\mathcal{S} \\
& \xi_{ij} - \sum_{s\in\mathcal{S}}\pi_{ij}^s + \theta_v \geq \sum_{s\in\mathcal{S}} N^s Q_{ij}^s x_{ij}^s && \forall (i,j)\in\mathcal{A} \\
& \eta_{ij}^s + \pi_{ij}^s \geq K^s Q_{ij}^s x_{ij}^s && \forall (i,j)\in\mathcal{A}, s\in\mathcal{S} \\
& \rho^s, \xi_{ij}, \eta_{ij}^s, \pi_{ij}^s, \theta_u, \theta_v \geq 0 && \forall (i,j)\in\mathcal{A}, s\in\mathcal{S}
\end{aligned}
$$

We present the single-level linear optimization problem for C-Master:

[C-Master]

$$
\begin{aligned}
\underset{\mathbf{x},\mathbf{y},\mathbf{z},\boldsymbol{\rho},\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{\pi},\theta_u,\theta_v}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}} F_i y_i + \sum_{(i,j)\in\mathcal{A}}\sum_{s\in\mathcal{S}} N^s R_{ij}^s x_{ij}^s + \sum_{s\in S}\rho_s + \sum_{(i,j)\in\mathcal{A}}\xi_{ij} + \Gamma_u\theta_u + \Gamma_v\theta_v \\
\text{subject to} \quad & \sum_{j:(i,j)\in\mathcal{A}} x_{ij}^s - \sum_{j:(j,i)\in\mathcal{A}} x_{ji}^s \begin{cases} = 1 & \text{if } i = o(s) \\ \geq -y_i & \text{if } i\in\mathcal{M} \\ = 0 & \text{otherwise} \end{cases} && \forall i\in\mathcal{N}, s\in\mathcal{S} && (8) \\
& x_{ij}^s \leq z_{ij} && \forall (i,j)\in\mathcal{A}, s\in\mathcal{S} && (9) \\
& x_{ij}^s \in \{0,1\} && \forall (i,j)\in\mathcal{A}, s\in\mathcal{S} && (10) \\
& y_i \in \{0,1\} && \forall i\in\mathcal{M} && (11) \\
& z_{ij} \in \{0,1\} && \forall (i,j)\in\mathcal{A} && (12) \\
& \rho^s - \sum_{(i,j)\in\mathcal{A}}\eta_{ij}^s + \theta_u \geq \sum_{(i,j)\in\mathcal{A}} K^s R_{ij} x_{ij}^s && \forall s\in\mathcal{S} && (13) \\
& \xi_{ij} - \sum_{s\in\mathcal{S}}\pi_{ij}^s + \theta_v \geq \sum_{s\in\mathcal{S}} N^s Q_{ij}^s x_{ij}^s && \forall (i,j)\in\mathcal{A} && (14) \\
& \eta_{ij}^s + \pi_{ij}^s \geq K^s Q_{ij}^s x_{ij}^s && \forall (i,j)\in\mathcal{A}, s\in\mathcal{S} && (15) \\
& \rho^s, \xi_{ij}, \eta_{ij}^s, \pi_{ij}^s, \theta_u, \theta_v \geq 0 && \forall (i,j)\in\mathcal{A}, s\in\mathcal{S} && (16) \\
& \text{additional cuts (23) (Section 4.2) added}
\end{aligned}
$$

Let $\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}}$ be the solution of the C-master problem. The C-master problem is still difficult to solve; we use Benders decomposition to solve it. C-Master is divided into an integer Benders Master problem (B-Master) and a continuous Benders Sub problem (B-Sub). The decision variables are divided into two parts: binary variables x_{ij}^s, y_i, z_{ij} and continuous variables $\rho^s, \xi_{ij}, \eta_{ij}^s, \pi_{ij}^s, \theta_u, \theta_v$. The B-Sub problem generates a cut that is added to B-Master problem in every iteration. When the objectives of B-Master and B-Sub are same, the solutions $\bar{\mathbf{x}}$, $\bar{\mathbf{y}}$, and $\bar{\mathbf{z}}$ are obtained.

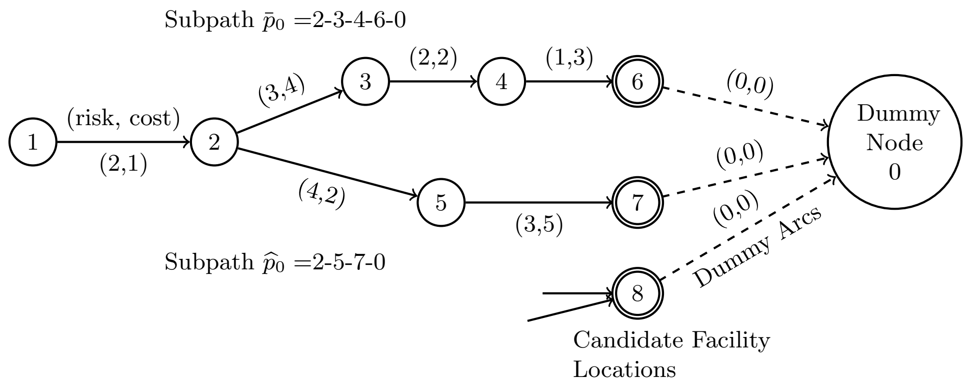


Figure 2: Conversion to a pure network design problem

Fixing $\mathbf{y} = \bar{\mathbf{y}}$ and $\mathbf{z} = \bar{\mathbf{z}}$, we write the C-Sub problem as follows:

[C-Sub]

$$\underset{\mathbf{x}}{\text{minimize}} \quad \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} c_{ij} x_{ij}^s$$

$$\text{subject to} \quad \sum_{j:(i,j)\in\mathcal{A}} x_{ij}^s - \sum_{j:(j,i)\in\mathcal{A}} x_{ji}^s \begin{cases} = 1 & \text{if } i = o(s) \\ \geq -\bar{y}_i & \text{if } i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} \qquad \forall i \in \mathcal{N}, s \in \mathcal{S}$$

$$x_{ij}^s \leq \bar{z}_{ij} \qquad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}$$

$$x_{ij}^s \in \{0,1\} \qquad \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}$$

Let $\widehat{\mathbf{x}}$ be the solution of the C-Sub problem, which represents the path which minimizes the transportation costs with the given facility location and network design.

4.2 Cut Generation

While Gzara (2013) has provided effective cut generation methods for the hazmat network design problem, our problem involves both network design and facility location variables. To apply the method of Gzara (2013) to our problem, we first transform the combined facility location and network design problem to a pure network design problem (Melkote and Daskin, 2001b). As shown in Figure 2, all facility candidate locations are first connected to a dummy node via dummy arcs. Since the risk and transportation cost are zero in all dummy arcs, constructing a facility in a candidate location is equivalent to opening the corresponding dummy arc for traveling. By adding the dummy node, labeled as ‘0’, and dummy arcs, labeled as $(k, 0)$ for each candidate location $k \in \mathcal{M}$, we obtain new sets of nodes and arcs as follows:

$$\mathcal{N}_0 = \mathcal{N} \cup \{0\}$$
$$\mathcal{A}_0 = \mathcal{A} \cup \{(k,0) : k \in \mathcal{M}\}$$

As a result, we obtain an augmented graph $G_0(\mathcal{N}_0, \mathcal{A}_0)$, in which new "network design" variable z_{k0} for each $k \in \mathcal{M}$ corresponds to location variable y_k.

For each shipment s, two solutions $\bar{\mathbf{x}}$ and $\widehat{\mathbf{x}}$ utilize different paths. Among such two different paths, we obtain two distinct subpaths $\bar{p}$ and $\widehat{p}$ from $\bar{\mathbf{x}}$ and $\widehat{\mathbf{x}}$, respectively. Adding the dummy node to $\bar{p}$ and $\widehat{p}$, we obtain subpaths $\bar{p}_0$ and $\widehat{p}_0$ defined in $G_0(\mathcal{N}_0, \mathcal{A}_0)$, respectively. When the cuts suggested by Gzara (2013) are applied in $G_0(\mathcal{N}_0, \mathcal{A}_0)$, we obtain:

$$\sum_{(i,j)\in\bar{p}_0} x^s_{ij} \le |\bar{p}_0| - 1 + u_{\text{new}} \tag{17}$$

$$u_{\text{new}} \le x^s_{ij} \qquad \forall (i,j) \in \bar{p}_0 \tag{18}$$

$$\sum_{(i,j)\in\widehat{p}_0} z_{ij} \le |\widehat{p}_0| - u_{\text{new}} \tag{19}$$

$$u_{\text{new}} \in \{0,1\} \tag{20}$$

where $|p|$ means the number of arcs in path p. We first show that the above cuts can be simplified.

Proposition 1. Inequalities in (17)–(20) hold if and only if

$$\sum_{(i,j)\in\widehat{p}_0} z_{ij} \le |\widehat{p}_0| + |\bar{p}_0| - 1 - \sum_{(i,j)\in\bar{p}_0} x^s_{ij} \tag{21}$$

holds.

Proof. We consider each direction separately.

[$\Longrightarrow$] Summing inequalities (17) and (19), we obtain

$$\sum_{(i,j)\in\bar{p}_0} x^s_{ij} + \sum_{(i,j)\in\widehat{p}_0} z_{ij} \le |\bar{p}_0| - 1 + |\widehat{p}_0|,$$

which is (21).

[$\Longleftarrow$] We now show that (21) implies (17)–(20). We consider two cases:

(i) When $\sum_{(i,j)\in\bar{p}_0} x^s_{ij} = |\bar{p}_0|$. Then $x^s_{ij} = 1$ for all $(i,j) \in \bar{p}_0$. Also (21) implies that

$$\sum_{(i,j)\in\widehat{p}_0} z_{ij} \le |\widehat{p}_0| - 1.$$

For such $\mathbf{x}$ and $\mathbf{z}$, we can set $u_{\text{new}} = 1$ so that (17)–(20) hold.

(ii) When $\sum_{(i,j)\in\bar{p}_0} x^s_{ij} \le |\bar{p}_0| - 1$. Then $x^s_{ij} = 0$ for some $(i,j) \in \bar{p}_0$. Observe that the right-hand-side of (21) is greater than or equals to $|\widehat{p}_0|$. Since z_{ij} is binary, we have $\sum_{(i,j)\in\widehat{p}_0} z_{ij} \le |\widehat{p}_0|$ by definition. Therefore by setting $u_{\text{new}} = 0$, we find that (17)–(20) hold.

□

Note that (21) can be written as

$$\sum_{(i,j)\in\widehat{p}_0}(1-z_{ij}) \geq 1-|\bar{p}_0| + \sum_{(i,j)\in\bar{p}_0} x^s_{ij}, \tag{22}$$

which has the following simple interpretation. If we want to flow $\mathbf{x}$ through subpath $\bar{p}_0$, i.e. $\sum_{(i,j)\in\bar{p}_0} x^s_{ij} = |\bar{p}_0|$, then at least one arc $(i,j)\in\widehat{p}_0$ must be closed or $\sum_{(i,j)\in\widehat{p}_0}(1-z_{ij})\geq 1$.

Then we write the cut (22) in the original network $G(\mathcal{N},\mathcal{A})$.

Proposition 2. Let

$$\widehat{\delta}_k = \begin{cases} 1 & \text{if } \widehat{p} \text{ includes node } k \\ 0 & \text{otherwise} \end{cases}$$

for each $k\in\mathcal{M}$. Then the cut in (22) is equivalently written as

$$\sum_{(i,j)\in\widehat{p}}(1-z_{ij}) + \sum_{k\in\mathcal{M}}\widehat{\delta}_k(1-y_k) \geq 1-|\bar{p}| + \sum_{(i,j)\in\bar{p}} x^s_{ij} \tag{23}$$

for the original network $G(\mathcal{N},\mathcal{A})$.

Proof. If subpath $\bar{p}$ includes any facility location, we observe that

$$|\bar{p}_0| = |\bar{p}|+1$$
$$\sum_{(i,j)\in\bar{p}_0} x^s_{ij} = \sum_{(i,j)\in\bar{p}} x^s_{ij} + 1$$

since every shipment must flow to the dummy node. If subpath $\bar{p}$ does not involve a facility location, then

$$|\bar{p}_0| = |\bar{p}|$$
$$\sum_{(i,j)\in\bar{p}_0} x^s_{ij} = \sum_{(i,j)\in\bar{p}} x^s_{ij}.$$

Therefore, in both cases, we have

$$|\bar{p}_0| - \sum_{(i,j)\in\bar{p}_0} x^s_{ij} = |\bar{p}| - \sum_{(i,j)\in\bar{p}} x^s_{ij}.$$

With similar consideration, we also observe that

$$\sum_{(i,j)\in\widehat{p}_0}(1-z_{ij}) = \sum_{(i,j)\in\widehat{p}}(1-z_{ij}) + \sum_{k\in\mathcal{M}}\widehat{\delta}_k(1-y_k).$$

Hence, we obtain a proof. □

4.3 Benders Decomposition for Solving C-Master

To solve the C-Master problem, we consider Benders Decomposition. The Benders Master (B-Master) problem contains binary variables x_{ij}^s, y_i, and z_{ij} and constraints that restrict the binary variables; namely, (8)–(12) and the cuts added in C-Master. We define the B-Master problem as follows:

[B-Master]

$$\begin{aligned}
\underset{\mathbf{x},\mathbf{y},\mathbf{z},d}{\text{minimize}} \quad & \sum_{i\in\mathcal{M}} F_i y_i + \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} N^s R_{ij}^s x_{ij}^s + d \\
\text{subject to} \quad & \sum_{j:(i,j)\in\mathcal{A}} x_{ij}^s - \sum_{j:(j,i)\in\mathcal{A}} x_{ji}^s \begin{cases} = 1 & \text{if } i = o(s) \\ \geq -y_i & \text{if } i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} && \forall i \in \mathcal{N}, s \in \mathcal{S} \\
& x_{ij}^s \leq z_{ij} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& x_{ij}^s \in \{0,1\} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& y_i \in \{0,1\} && \forall i \in \mathcal{M} \\
& z_{ij} \in \{0,1\} && \forall (i,j) \in \mathcal{A} \\
& \text{cuts (23) (Section 4.2) carried over from C-Master} \\
& \text{additional Benders cuts (24) added}
\end{aligned}$$

where d represents the remainder of the objective function that will be computed by sub-problems and constrained by Benders cuts. Let $\widetilde{\mathbf{x}}$, $\widetilde{\mathbf{y}}$, $\widetilde{\mathbf{z}}$, and $\widetilde{d}$ denote the optimal solutions of B-Master. Then B-Master gives a lower bound for C-Master. We let

$$\mathsf{LB} = \sum_{i\in\mathcal{M}} F_i \widetilde{y}_i + \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} N^s R_{ij}^s \widetilde{x}_{ij}^s + \widetilde{d}.$$

The Benders Sub (B-Sub) problem contains continuous variables $\boldsymbol{\rho}, \boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\pi}, \theta_u, \theta_v$ and constraints (13)–(16). The B-Sub problem is given by fixing $\mathbf{x}$, $\mathbf{y}$, and $\mathbf{z}$ with a solution of $\widetilde{\mathbf{x}}$, $\widetilde{\mathbf{y}}$, and $\widetilde{\mathbf{z}}$ by solving the B-Master problem. The B-Sub problem is defined as follows:

[B-Sub]

$$\begin{aligned}
\underset{\boldsymbol{\rho},\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{\pi},\theta_u,\theta_v}{\text{minimize}} \quad & \left[\sum_{s\in S} \rho_s + \sum_{(i,j)\in\mathcal{A}} \xi_{ij} + \Gamma_u \theta_u + \Gamma_v \theta_v \right] \\
\text{subject to} \quad & \rho^s - \sum_{(i,j)\in\mathcal{A}} \eta_{ij}^s + \theta_u \geq \sum_{(i,j)\in\mathcal{A}} K^s R_{ij} \widetilde{x}_{ij}^s \qquad \forall s \in \mathcal{S} && (\alpha^s)
\end{aligned}$$

$$\xi_{ij} - \sum_{s\in\mathcal{S}} \pi^s_{ij} + \theta_v \geq \sum_{s\in\mathcal{S}} N^s Q^s_{ij}\widetilde{x}^s_{ij} \qquad \forall (i,j)\in\mathcal{A} \qquad (\beta_{ij})$$

$$\eta^s_{ij} + \pi^s_{ij} \geq K^s Q^s_{ij}\widetilde{x}^s_{ij} \qquad \forall (i,j)\in\mathcal{A}, s\in\mathcal{S} \qquad (\gamma^s_{ij})$$

$$\rho^s, \xi_{ij}, \eta^s_{ij}, \pi^s_{ij}, \theta_u, \theta_v \geq 0 \qquad \forall (i,j)\in\mathcal{A}, s\in\mathcal{S}$$

Because the optimality and valid cuts of B-Master problem can be defined by the dual variables of B-Sub problem, we formulate the dual for B-Sub. The dual variables α^s, β_{ij}, and γ^s_{ij} are introduced. The dual problem for B-Sub is presented as follows:

[The Dual of B-Sub]

$$\begin{aligned}
\underset{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma}}{\text{maximize}} \quad & \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} (K^s R^s_{ij}\widetilde{x}^s_{ij}\alpha^s + N^s Q^s_{ij}\widetilde{x}^s_{ij}\beta_{ij} + K^s Q^s_{ij}\widetilde{x}^s_{ij}\gamma^s_{ij}) \\
\text{subject to} \quad & \alpha^s \leq 1 && \forall s\in\mathcal{S} \\
& \beta_{ij} \leq 1 && \forall (i,j)\in\mathcal{A} \\
& -\alpha^s + \gamma^s_{ij} \leq 0 && \forall (i,j)\in\mathcal{A}, s\in\mathcal{S} \\
& -\beta_{ij} + \gamma^s_{ij} \leq 0 && \forall (i,j)\in\mathcal{A}, s\in\mathcal{S} \\
& \sum_{s\in\mathcal{S}} \alpha^s \leq \Gamma_u \\
& \sum_{(i,j)\in\mathcal{A}} \beta_{ij} \leq \Gamma_v \\
& \alpha^s, \beta_{ij}, \gamma^s_{ij} \geq 0 && \forall (i,j)\in\mathcal{A}, s\in\mathcal{S}
\end{aligned}$$

Let $\widetilde{\alpha}^s$, $\widetilde{\beta}_{ij}$, and $\widetilde{\gamma}^s_{ij}$ be the optimal solution of the Dual of the B-Sub problem. The following valid cut is added to the B-Master problem:

$$\sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} (K^s R^s_{ij}\widetilde{\alpha}^s + N^s Q^s_{ij}\widetilde{\beta}_{ij} + K^s Q^s_{ij}\widetilde{\gamma}^s_{ij}) x^s_{ij} \leq d. \tag{24}$$

We also obtain an upper bound for C-Master as follows:

$$\mathsf{UB} = \sum_{i\in\mathcal{M}} F_i\widetilde{y}_i + \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} N^s R^s_{ij}\widetilde{x}^s_{ij} + \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} (K^s R^s_{ij}\widetilde{x}^s_{ij}\widetilde{\alpha}^s + N^s Q^s_{ij}\widetilde{x}^s_{ij}\widetilde{\beta}_{ij} + K^s Q^s_{ij}\widetilde{x}^s_{ij}\widetilde{\gamma}^s_{ij}).$$

If $\mathsf{UB} = \mathsf{LB}$, then an optimal solution for C-Master is obtained.

5 A Single-Level Reformulation

We provide a single-level reformulation of the robust combined location-network design problem given in (1)–(7). We first replace the lower-level problem by its optimality conditions using techniques similar to the methods used by Arslan et al. (2018). Then we dualize and linearize the inner maximization problem for the worst-case consideration as done in Berglund and Kwon (2014).

The resulting single-level reformulation involves a big-M like constant bounded by $\sum_{(i,j)\in\mathcal{A}} c_{ij}$. We will use this single-level reformulation as a benchmark for the cutting-plane method developed in Section 4.

5.1 Replacing the Lower-Level Problem by Optimality Conditions

Since the lower-level shortest path problem has the property of totally unimodular matrices (Kara and Verter, 2004), the binary variable x_{ij}^s can be relaxed to a nonnegative real number. We also introduce a dummy node 0 to transform the problem into a pure network design problem as done in Section 4.2. The lower-level problem can be written equivalently as follows:

$$
\begin{aligned}
\underset{\mathbf{x}}{\text{minimize}} \quad & \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} c_{ij} x_{ij}^s \\
\text{subject to} \quad & \sum_{j:(i,j)\in\mathcal{A}} x_{ij}^s - \sum_{j:(j,i)\in\mathcal{A}} x_{ji}^s \begin{cases} = 1 & \text{if } i = o(s) \\ +x_{i0} = 0 & \text{if } i \in \mathcal{M} \\ = -1 & \text{if } i = 0 \\ = 0 & \text{otherwise} \end{cases} && \forall i \in \mathcal{N}, s \in \mathcal{S} && (-\lambda_i^s) \\
& (1 - z_{ij}) x_{ij}^s \le 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} && (-\mu_{ij}^s) \\
& (1 - y_i) x_{i0}^s \le 0 && \forall i \in \mathcal{M}, s \in \mathcal{S} && (-\mu_{i0}^s) \\
& x_{ij}^s \ge 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& x_{i0}^s \ge 0 && \forall i \in \mathcal{M}, s \in \mathcal{S}
\end{aligned}
$$

The dual variables $-\lambda_i^s$ and $-\mu_{ij}^s$ are introduced. The dual problem is:

$$
\begin{aligned}
\underset{\boldsymbol{\lambda},\boldsymbol{\mu}}{\text{maximize}} \quad & \sum_{s\in\mathcal{S}} \left(\lambda_0^s - \lambda_{o(s)}^s\right) && && (25) \\
\text{subject to} \quad & -\lambda_i^s + \lambda_j^s - (1 - z_{ij})\mu_{ij}^s \le c_{ij} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} && (26) \\
& -\lambda_i^s + \lambda_0^s - (1 - y_i)\mu_{i0}^s \le 0 && \forall i \in \mathcal{M}, \forall s \in \mathcal{S} && (27) \\
& \mu_{ij}^s \ge 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} && (28) \\
& \mu_{i0}^s \ge 0 && \forall i \in \mathcal{M}, s \in \mathcal{S} && (29)
\end{aligned}
$$

Using an approach similar to Arslan et al. (2018), we obtain the following result:

Proposition 3. Let $\bar{\mu} = \sum_{(i,j)\mathcal{A}} c_{ij}$. There exists an optimal solution for (25)–(29) with $\mu_{ij}^s = \mu_{i0}^s = \bar{\mu}$ for all $s \in \mathcal{S}$, $(i,j) \in \mathcal{A}$ and $i \in \mathcal{M}$.

Proof. By letting $\lambda_{o(s)}^s = 0$ without loss of generality, we obtain:

$$
\underset{\boldsymbol{\lambda},\boldsymbol{\mu}}{\text{maximize}} \quad \sum_{s\in\mathcal{S}} \lambda_0^s
$$

$$
\begin{aligned}
\text{subject to } & \lambda_j^s \le \lambda_i^s + c_{ij} + (1 - z_{ij})\mu_{ij}^s && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \lambda_0^s \le \lambda_i^s + (1 - y_i)\mu_{i0}^s && \forall i \in \mathcal{M}, \forall s \in \mathcal{S} \\
& \mu_{ij}^s \ge 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \mu_{i0}^s \ge 0 && \forall i \in \mathcal{M}, s \in \mathcal{S}
\end{aligned}
$$

Note that $\boldsymbol{\mu}$ does not contribute to the objective function; therefore we can make $(1 - z_{ij})\mu_{ij}^s$ and $(1 - y_i)\mu_{i0}^s$ arbitrarily large to maximize λ_0^s. Since λ_0^s represents a label for node d, we can bound μ_{ij}^s and μ_{i0}^s by $\bar{\mu}$. □

Therefore, the dual feasibility becomes:

$$
\begin{aligned}
& \lambda_j^s \le \lambda_i^s + c_{ij} + (1 - z_{ij})\bar{\mu} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \lambda_0^s \le \lambda_i^s + (1 - y_i)\bar{\mu} && \forall i \in \mathcal{M}, \forall s \in \mathcal{S}.
\end{aligned}
$$

Note that $\bar{\mu}$ behaves like big-M constants. For the optimality condition, instead of the strong duality, we can use the reverse weak duality (Amaldi et al., 2011; Arslan et al., 2018) in the following form:

$$
\sum_{s \in \mathcal{S}} \left(\lambda_0^s - \lambda_{o(s)}^s\right) \ge \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} c_{ij} x_{ij}^s
$$

Therefore, the robust combined location-network design problem becomes:

$$
\begin{aligned}
\underset{\mathbf{x},\mathbf{y},\mathbf{z},\boldsymbol{\lambda}}{\text{minimize}} \quad & \left[\sum_{i \in \mathcal{M}} F_i y_i + \max_{\mathbf{u} \in \mathbf{U}, \mathbf{v} \in \mathbf{V}} \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} (N^s + K^s u^s)(R_{ij}^s + Q_{ij}^s v_{ij}) x_{ij}^s \right] \\
\text{subject to} \quad & \sum_{j:(i,j) \in \mathcal{A}} x_{ij}^s - \sum_{j:(j,i) \in \mathcal{A}} x_{ji}^s \begin{cases} = 1 & \text{if } i = o(s) \\ \ge -y_i & \text{if } i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} && \forall i \in \mathcal{N}, s \in \mathcal{S} \\
& x_{ij}^s \le z_{ij} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \sum_{s \in \mathcal{S}} \left(\lambda_0^s - \lambda_{o(s)}^s\right) \ge \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} c_{ij} x_{ij}^s \\
& \lambda_j^s \le \lambda_i^s + c_{ij} + (1 - z_{ij})\bar{\mu} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \lambda_0^s \le \lambda_i^s + (1 - y_i)\bar{\mu} && \forall i \in \mathcal{M}, \forall s \in \mathcal{S} \\
& x_{ij}^s \in \{0,1\} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& y_i \in \{0,1\} && \forall i \in \mathcal{M} \\
& z_{ij} \in \{0,1\} && \forall (i,j) \in \mathcal{A} \\
& \lambda_i^s \ge 0 && \forall i \in \mathcal{N} \cup \{0\}.
\end{aligned}
$$

5.2 Dualizing and Linearizing the Inner Maximization Problem

The inner maximization problem can be dualized and linearized as done in Section 4.1. Finally, we obtain the single-level reformulation of the robust combined location-network design problem as follows:

$$
\begin{aligned}
\underset{\mathbf{x},\mathbf{y},\mathbf{z},\boldsymbol{\lambda},\boldsymbol{\rho},\boldsymbol{\xi},\boldsymbol{\eta},\boldsymbol{\pi},\theta_u,\theta_v}{\text{minimize}} \quad & \Bigg[\sum_{i\in\mathcal{M}} F_i y_i + \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} N^s R_{ij}^s x_{ij}^s + \sum_{s\in S} \rho^s + \sum_{(i,j)\in\mathcal{A}} \xi_{ij} + \Gamma_u \theta_u + \Gamma_v \theta_v \Bigg] \\
\text{subject to} \quad & \sum_{j:(i,j)\in\mathcal{A}} x_{ij}^s - \sum_{j:(j,i)\in\mathcal{A}} x_{ji}^s \begin{cases} = 1 & \text{if } i = o(s) \\ \geq -y_i & \text{if } i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} && \forall i \in \mathcal{N}, s \in \mathcal{S} \\
& x_{ij}^s \leq z_{ij} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \sum_{s\in\mathcal{S}} \left(\lambda_0^s - \lambda_{o(s)}^s \right) \geq \sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} c_{ij} x_{ij}^s \\
& \lambda_j^s \leq \lambda_i^s + c_{ij} + (1 - z_{ij})\bar{\mu} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \lambda_0^s \leq \lambda_i^s + (1 - y_i)\bar{\mu} && \forall i \in \mathcal{M}, \forall s \in \mathcal{S} \\
& \rho^s - \sum_{(i,j)\in\mathcal{A}} \eta_{ij}^s + \theta_u \geq \sum_{(i,j)\in\mathcal{A}} K^s R_{ij}^s x_{ij}^s && \forall s \in \mathcal{S} \\
& \xi_{ij} - \sum_{s\in\mathcal{S}} \pi_{ij}^s + \theta_v \geq \sum_{s\in\mathcal{S}} N^s Q_{ij}^s x_{ij}^s && \forall (i,j) \in \mathcal{A} \\
& \eta_{ij}^s + \pi_{ij}^s \geq K^s Q_{ij}^s x_{ij}^s && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& x_{ij}^s \in \{0,1\} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& y_i \in \{0,1\} && \forall i \in \mathcal{M} \\
& z_{ij} \in \{0,1\} && \forall (i,j) \in \mathcal{A} \\
& \lambda_i^s \geq 0 && \forall i \in \mathcal{N} \cup \{d\} \\
& \rho^s, \xi_{ij}, \eta_{ij}^s, \pi_{ij}^s, \theta_u, \theta_v \geq 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}
\end{aligned}
$$

The above problem is a mixed integer linear program (MILP) where all integer variables are binary. Off-the-shelf optimization solvers such as Gurobi and CPLEX can be used to solve small-size problem. As the size increases, however, the amount of time required by solvers grows rapidly.

6 Numerical Experiments

The experiments are done on the computer which runs 64-bit Windows 10 with 2.60GHz Intel Core (TM i5-7300U) CPU and 8 GB RAM. The cutting plane algorithm is coded in Julia 0.6.4 (Bezanson et al., 2012) and JuMP.jl optimization modeling package (Dunning et al., 2017) is used. The single-level reformulation in Section 5 is solved by calling Gurobi 7.5.2 solver with default setting.

Numerical analysis is performed on a set of data from Ravenna city in Italy (Erkut and Alp, 2007). The road network in Ravenna is consists of 111 nodes and 143 arcs. The risk R_{ij}^s on arc (i, j) is calculated as the summation of exposure risk from all four types of hazmat (methanol, chlorine, gasoline, and LPG). The transportation cost c_{ij} for arc (i, j) is measured as the actual distance in meters. The demand for each shipment s is measured as truckloads, i.e., the number of trucks N^s.

The experiments are done on two sets of instances, small size and large size. In the small-size problems, there are 9 origins and the set size of candidate facility locations $\mathcal{M}$ is 5 and 10. In the large-size problems, there are 20 origins of hazmat shipment. We randomly choose 5, 10 and 15 as the set of candidate facility locations $\mathcal{M}$.

The comparison between objectives and running time of the cutting plane algorithm and Gurobi for the single-level reformulation are calculated as follows:

$$\%\text{Obj} = \frac{\text{Objective of Gurobi} - \text{Objective of cutting plane}}{\text{Objective of cutting plane}} \times 100 \tag{30}$$

$$\%\text{Time} = \frac{\text{Running time of Gurobi} - \text{Running time of cutting plane}}{\text{Running time of cutting plane}} \times 100 \tag{31}$$

6.1 Analysis on the Small-Size Instances

The objectives and running time on the small-size instances are shown in Table 2. For instances 3, 4, and 9, Gurobi fails to obtain a proven optimal solution in 3600s. The gaps between the incumbent solution and the best bound are 1.27%, 2.13%, and 0.71%, respectively. The cutting plane algorithm can take less running time to get the proven optimal solutions except instances 5, 10, 15, and 20. Γ_u and Γ_v are much larger than those in other instances. As a result, the worst-case in the inner maximization problem happens when almost all u and v variables are set to 1. This makes the problem easier to solve for larger Γ_u and Γ_v values. So, for these instances, the optimal solution can be obtained in less running time by Gurobi than the cutting plane algorithm. The cutting plane algorithm performs better than Gurobi on 80% small-size instances in terms of running time. The average %Obj is 0.00% and the average %Time is 476.37%. The cutting plane algorithm and Gurobi can get optimal solution values for all small-size instances.

The results for Instance 1 and 11 are illustrated in Figure 3. The circles denote origins where hazmat is generated. The triangles represent chosen facilities sites. The green lines denote the routes which truck drivers choose. The red lines denote the roads which are not available for hazmat transportation.

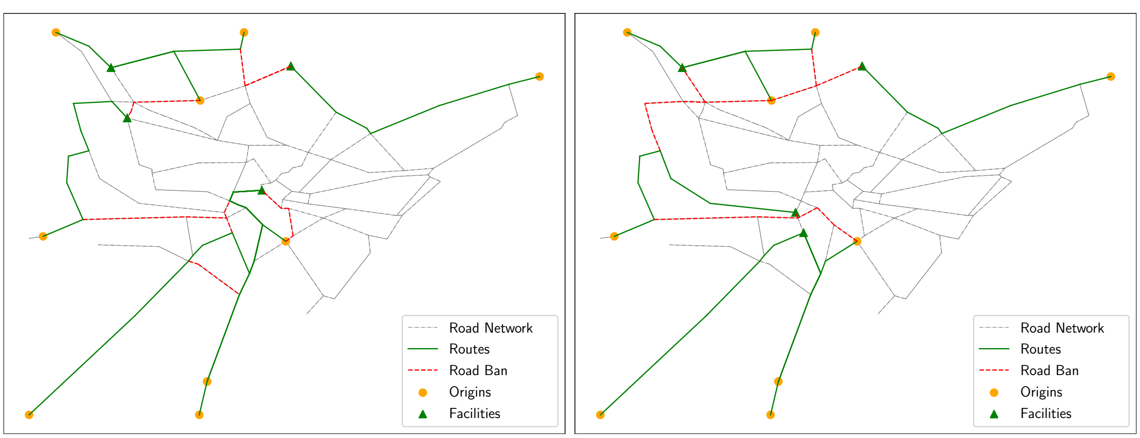
6.2 Analysis on the Large-Size Instances

The objectives and running time on the large-size instances are shown in Table 3. The cutting plane algorithm and Gurobi can obtain optimal solutions for 100% and 58.33% large-size instances in 10800s, respectively. For instances 2 and 11, Gurobi obtains the incumbent solution that is equal to the value of the optimal solution. But the optimality of the incumbent solution can't be proven. The gaps between the incumbent solution and the best bound are 0.16% and 0.66%, respectively.

Table 2: Comparison Between the Solutions by the Cutting Plane Algorithm and Gurobi for the Single-Level Reformulation on Small-Size Ravenna Instances

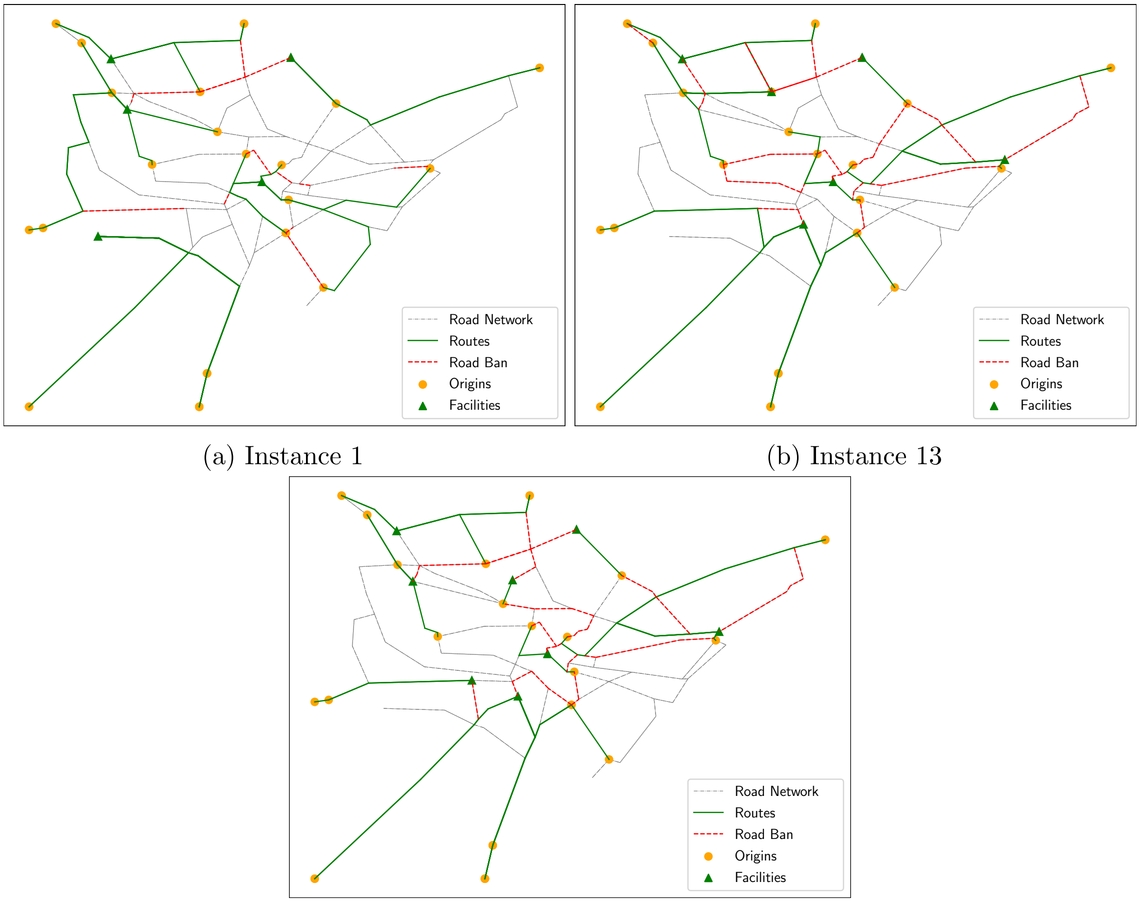
Instance				Cutting-Plane		Single-Level		Comparison	
No.	$\lvert\mathcal{M}\rvert$	(K^s, Q^s_{ij})	(Γ_u, Γ_v)	Objective	Time(s)	Objective	Time(s)	%Obj	%Time
1	5	(N^s, R^s_{ij})	$(1, 1)$	19390	1.2	19390	2.9	0.00	141.7
2			$(3, 5)$	28966	43.0	28966	495.8	0.00	1053.0
3			$(5, 5)$	31222	66.5	31222[a]	3600.0	0.00	5313.5
4			$(5, 10)$	36221	1764.5	36221[a]	3600.0	0.00	104.0
5			$(10, 20)$	41814	1559.8	41814	655.4	0.00	-58.0
6		$(0.5N^s, 0.5R^s_{ij})$	$(1, 1)$	15628	1.1	15628	1.7	0.00	54.5
7			$(3, 5)$	19783	7.5	19783	36.8	0.00	390.7
8			$(5, 5)$	20779	11.3	20779	94.1	0.00	732.7
9			$(5, 10)$	22754	221.1	22754[a]	3600.0	0.00	1528.2
10			$(10, 20)$	24893	242.5	24893	96.3	0.00	-60.3
11	10	(N^s, R^s_{ij})	$(1, 1)$	11783	1.4	11783	1.9	0.00	35.7
12			$(3, 5)$	19491	2.6	19491	4.7	0.00	80.8
13			$(5, 5)$	20377	3.4	20377	5.3	0.00	55.9
14			$(5, 10)$	22678	5.0	22678	5.3	0.00	6.0
15			$(10, 20)$	25470	29.4	25470	6.0	0.00	-79.6
16		$(0.5N^s, 0.5R^s_{ij})$	$(1, 1)$	9424	1.3	9424	1.4	0.00	7.7
17			$(3, 5)$	12770	1.3	12770	1.6	0.00	23.1
18			$(5, 5)$	13158	1.4	13158	5.4	0.00	285.7
19			$(5, 10)$	13953	1.4	13953	1.4	0.00	0.0
20			$(10, 20)$	15251	20.1	15251	2.4	0.00	-88.1
Average								**0.00**	**476.37**

[a] The algorithm stopped in 3600s and the solution is not proven optimal.



(a) Instance 1 (b) Instance 11

Figure 3: Results on Small-Size Ravenna Instances



(a) Instance 1

(b) Instance 13

(c) Instance 25

Figure 4: Results on Large-Size Ravenna Instances

For all instances, the cutting plane algorithm can take much less running time to get the proven optimal solutions. The average %Obj is 5.00% and the average %Time is 820.32%. The cutting plane algorithm outperforms the single-level reformulation solved by Gurobi in solution quality and running time. The results for Instance 1, 13, and 25 are illustrated in Figure 4.

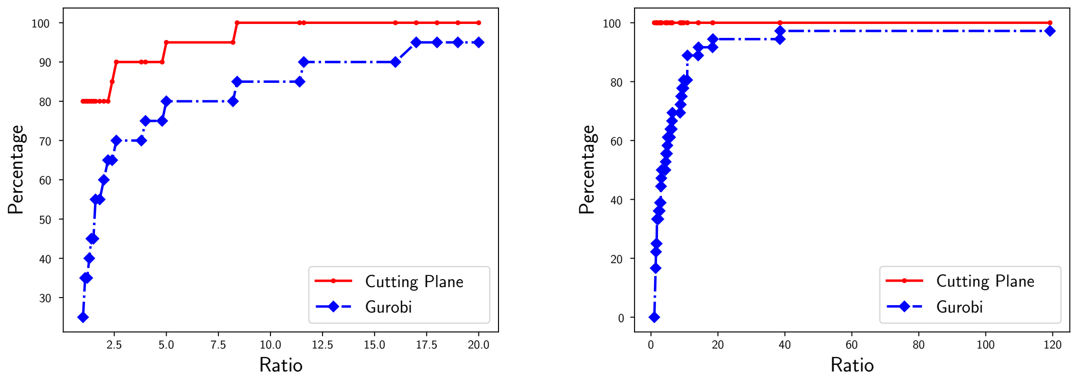
The performance profile (Dolan and Moré, 2002) is used to compare different algorithms on the running times. The running time performance file for different algorithms is created by calculating the ratios of the running time of each algorithm and the minimum running time of all algorithms. The horizontal axis shows the ratios. The vertical axis shows the percentage of instances with a ratio that is less than or equal to the ratio on the horizontal axis. This indicates that the method has better performance when its profile is drawn in the upper left-hand graph. The running time performance profiles of the proposed cutting plane algorithm and Gurobi for the single-level reformation on small-size and large-size Ravenna instances are shown in Figure 5.

Figure 5 indicates that the cutting plane algorithm has better running time performance than the single-level reformulation solved by Gurobi on small-size and large-size Ravenna Instances. In Figure 5(b), the profile of large-size instances is a straight line, that means the running times of

Table 3: Comparison Between the Solutions by the Cutting Plane Algorithm and Gurobi for the Single-Level Reformulation on Large-Size Ravenna Instances

Instance				Cutting-Plane		Single-Level		Comparison	
No.	$\|\mathcal{M}\|$	(K^s, Q_{ij}^s)	(Γ_u, Γ_v)	Objective	Time(s)	Objective	Time(s)	%Obj	%Time
1	5	(N^s, R_{ij}^s)	$(1,1)$	90089	8.4	90089	155.4	0.00	1750.00
2			$(3,5)$	137010	279.8	137010[a]	10800.0	0.00	3759.90
3			$(5,5)$	143015	41.9	143015	389.0	0.00	828.40
4			$(5,10)$	168512	148.4	168512	717.3	0.00	383.36
5			$(10,20)$	200155	4832.4	200296[a]	10800.0	0.10	123.49
6			$(20,20)$	266569	652.0	266569	893.6	0.00	37.06
7		$(0.5N^s, 0.5R_{ij}^s)$	$(1,1)$	74187	12.5	74187	38.0	0.00	204.00
8			$(3,5)$	93607	22.6	93607	2695.3	0.00	11826.11
9			$(5,5)$	96609	13.8	96609	194.8	0.00	1311.59
10			$(5,10)$	106437	46.3	106437	122.1	0.00	163.71
11			$(10,20)$	119942	1233.6	119942[a]	10800.0	0.00	775.49
12			$(20,20)$	124410	994.2	124660[a]	10800.0	0.20	986.30
13	10	(N^s, R_{ij}^s)	$(1,1)$	50908	993.0	59432[a]	10800.0	16.74	987.61
14			$(3,5)$	67665	1723.9	85216[a]	10800.0	25.94	526.49
15			$(5,5)$	71346	1878.0	89698[a]	10800.0	25.72	475.08
16			$(5,10)$	80241	1111.5	97450[a]	10800.0	21.45	871.66
17			$(10,20)$	95987	3621.7	98472[a]	10800.0	2.59	198.20
18			$(20,20)$	104626	1804.6	104626	2485.5	0.00	37.73
19		$(0.5N^s, 0.5R_{ij}^s)$	$(1,1)$	39968	1203.2	46563[a]	10800.0	16.50	797.61
20			$(3,5)$	46879	1704.6	56822[a]	10800.0	21.21	533.58
21			$(5,5)$	48655	2464.6	58829[a]	10800.0	20.91	338.20
22			$(5,10)$	52257	998.5	62170[a]	10800.0	18.97	981.62
23			$(10,20)$	58732	3709.4	63444[a]	10800.0	8.02	191.15
24			$(20,20)$	72117	1406.2	73233[a]	10800.0	1.55	348.84
25	15	(N^s, R_{ij}^s)	$(1,1)$	48717	3.4	48717	16.8	0.00	394.12
26			$(3,5)$	65921	66.2	65921	100.8	0.00	52.27
27			$(5,5)$	70251	128.2	70251	223.7	0.00	74.49
28			$(5,10)$	77221	14.1	77221	19.8	0.00	40.43
29			$(10,20)$	92991	136.9	92991	175.2	0.00	27.98
30			$(20,20)$	100673	129.1	100673	225.4	0.00	74.59
31		$(0.5N^s, 0.5R_{ij}^s)$	$(1,1)$	38090	45.1	38090	66.7	0.00	47.89
32			$(3,5)$	45218	225.6	45218	713.0	0.00	216.05
33			$(5,5)$	47211	166.5	47211	204.5	0.00	22.82
34			$(5,10)$	50117	78.1	50117	134.7	0.00	72.47
35			$(10,20)$	56673	31.6	56673	42.4	0.00	34.18
36			$(20,20)$	59772	26.4	59772	36.2	0.00	37.12
Average								**5.00**	**820.32**

[a] The algorithm stopped in 10800s and the solution is not proven optimal.



(a) Small-Size Ravenna Instances (b) Large-Size Ravenna Instances

Figure 5: The Running Time Performance Profile of the Cutting Plane Algorithm and Gurobi for the Single-Level Reformulation

the cutting plane algorithm are less than those by Gurobi for all instances. Compared large-size instances 5, 11, 17, and 23 with small-size instances 5, 10, 15, and 20, the difficulty caused by increasing problem size is more than the simplicity caused by large Γ_u and Γ_v values. Besides, the cutting plane algorithm takes less time in large-size instances 6, 12, 18, 24, 30, and 36, which have larger parameters values $(\Gamma_u, \Gamma_v) = (20, 20)$. Therefore, the advantage of the cutting plane algorithm becomes obvious as the problem size increases, especially in the running time.

6.3 Combined Model versus Sequential Model

We consider the *combined* robust model that optimizes facility locations and network design problem simultaneously. If we use techniques proposed in the previous papers in the literature review, the problem has to be solved in two phases. In Phase 1, robust facility locations problem is solved, and facility setup locations are determined. In Phase 2, robust network design problem is solved. We call this two-phase model the *sequential* model.

In this paper, we use the single-level reformulation of bi-level robust facility location problem proposed by Kwon et al. (2013), shown in Appendix A. We obtain the optimal solution of facility location variables $\mathbf{y}^*$. When $y_i^* = 1$ for $i \in \mathcal{M}$, facility i is chosen to open. We let $\mathcal{K}$ be the set of chosen facility locations. Sun et al. (2015) considered a robust network design problem only with the uncertainty in exposure risk. Based on their model, we revise a single-level form of robust network design model that considers uncertainty both in exposure risk and the number of shipment demand, shown in Appendix B. We obtain the optimal solution of routing variables $\mathbf{x}^*$ and network design variable $\mathbf{z}^*$.

To prove the benefits of the combined model, we compare it with the sequential model in the terms of objective function value $\sum_{i \in \mathcal{M}} F_i y_i^* + \sum_{(i,j) \in \mathcal{A}} \sum_{s \in \mathcal{S}} N^s R_{ij}^s x_{ij}^{s*} + \sum_{s \in S} \rho^{s*} + \sum_{(i,j) \in \mathcal{A}} \xi_{ij}^* + \Gamma_u \theta_u^* + \Gamma_v \theta_v^*$. The comparison between objectives for combined model and sequential model is calculated as follows:

$$\%\text{Deviation} = \frac{\text{Objective of Sequential Model} - \text{Objective of Combined Model}}{\text{Objective of Combined Model}} \times 100 \tag{32}$$

The single-level reformulation models in Appendices A and B are solved by calling Gurobi 7.5.2 solver with default setting. Big-M is set as a constant bounded by $\sum_{(i,j)\in\mathcal{A}} c_{ij}$.

The objectives of sequential model and %Deviation are shown in Tables 4 and 5. For small-size and large-size Ravenna city instances, the average %Deviation is 2.91 and 3.87, respectively. The objectives of the combined model are less than those of sequential model and the difference can be as large as 10.39%. In Table 5, we can observe that the difference is more obvious when $|\mathcal{M}|$ is smaller. When $|\mathcal{M}|$ is 5, 10, and 15, the average %Deviation is 6.52, 2.36, and 2.73, respectively.

This result indicates that when there are fewer choices of potential facility candidates, the value of combined decision-making becomes more significant. When there are more choices available for locations, there may exist a location that leads to safe network even without network design policy. On the other hand, with fewer choices available for locations, such a favorable option may be unavailable; hence one needs to consider both location and network design decisions at the same time. When considering facility location and network design problem jointly, the leader can make a better decision with the aim of reducing the facility setup costs and hazmat exposure risk.

7 Concluding Remarks

In this paper, a leader-follower decision problem is considered in the form of bi-level optimization. In the upper level, the leader aims to minimize the total facility construction costs and hazmat exposure risks by determining facilities locations and available roads for hazmat transportation. The leader effects the followers who intend to minimize their transportation costs when designing the road network. We apply a robust optimization approach to deal with the uncertainty in the exposure risk and the demand. A bi-level integer programming model is formulated where the upper level is a min-max problem and the lower level is a shortest-path problem. We devise an exact algorithm that combines a cutting plane algorithm with Benders decomposition and derive a single-level reformulation. Comparisons between two approaches are made on the Ravenna city data, in terms of objectives and the running time. The analysis on small and large size instances demonstrates that the proposed cutting plane algorithm performs much better than Gurobi as the problem size increases. The proposed cutting plane algorithm is an effective exact method for solving the robust combined facility location-network design problem.

A couple of directions for future research are suggested. First, uncertainty on origin locations can be considered. In this paper, we assumed that all origin nodes are exactly known. Since the hazmat facility location problem is for long-term decision, considering new hazmat origins in the future will lead to an important problem. Second, hazmat trips to locations other than the hazmat facilities can be incorporated. Although we considered hazmat trips to hazmat facilities only in this paper, there are also hazmat trips to other destinations. Hazmat network design policies will certainly impact

Table 4: Comparison Between the Objectives for Combined and Sequential Model on Small-Size Ravenna Instances

Instance				Objective		
No.	$\|\mathcal{M}\|$	(K^s, Q^s_{ij})	(Γ_u, Γ_v)	Combined Model	Sequential Model	%Deviation
1	5	(N^s, R^s_{ij})	$(1,1)$	19390	20417	5.30
2			$(3,5)$	28966	30720	6.06
3			$(5,5)$	31222	33193	6.31
4			$(5,10)$	36221	36764	1.50
5			$(10,20)$	41814	42591	1.86
6		$(0.5N^s, 0.5R^s_{ij})$	$(1,1)$	15628	15959	2.12
7			$(3,5)$	19783	20391	3.07
8			$(5,5)$	20779	21451	3.23
9			$(5,10)$	22754	22791	0.16
10			$(10,20)$	24893	25072	0.72
11	10	(N^s, R^s_{ij})	$(1,1)$	11783	12171	3.29
12			$(3,5)$	19491	20709	6.25
13			$(5,5)$	20377	21594	5.97
14			$(5,10)$	22678	23818	5.03
15			$(10,20)$	25470	25817	1.36
16		$(0.5N^s, 0.5R^s_{ij})$	$(1,1)$	9424	9549	1.33
17			$(3,5)$	12855	12770	0.67
18			$(5,5)$	13158	13243	0.65
19			$(5,10)$	13953	13964	0.08
20			$(10,20)$	15251	15758	3.32
Average						**2.91**

Table 5: Comparison Between the Objectives for Combined and Sequential Model on Large-Size Ravenna Instances

Instance				Objective		
No.	$\|\mathcal{M}\|$	(K^s, Q^s_{ij})	(Γ_u, Γ_v)	Combined Model	Sequential Model	%Deviation
1	5	(N^s, R^s_{ij})	$(1,1)$	90089	99445	10.39
2			$(3,5)$	137010	148392	8.31
3			$(5,5)$	143015	157416	10.07
4			$(5,10)$	168512	176718	4.87
5			$(10,20)$	200155	211719	5.80
6			$(20,20)$	266569	273800	2.71
7		$(0.5N^s, 0.5R^s_{ij})$	$(1,1)$	74187	79259	6.84
8			$(3,5)$	93607	100067	6.90
9			$(5,5)$	96609	103821	7.47
10			$(5,10)$	106437	111429	4.69
11			$(10,20)$	119942	125868	4.94
12			$(20,20)$	124410	130981	5.28
13	10	(N^s, R^s_{ij})	$(1,1)$	50908	51377	0.92
14			$(3,5)$	67665	68255	0.87
15			$(5,5)$	71346	72974	2.28
16			$(5,10)$	80241	80628	0.48
17			$(10,20)$	95987	100021	4.20
18			$(20,20)$	104626	109641	4.79
19		$(0.5N^s, 0.5R^s_{ij})$	$(1,1)$	39968	40750	1.96
20			$(3,5)$	46879	47554	1.44
21			$(5,5)$	48655	49624	1.99
22			$(5,10)$	52257	52940	1.31
23			$(10,20)$	58732	61058	3.96
24			$(20,20)$	72117	75046	4.06
25	15	(N^s, R^s_{ij})	$(1,1)$	48717	50474	3.61
26			$(3,5)$	65921	66827	1.37
27			$(5,5)$	70251	71624	1.95
28			$(5,10)$	77221	78262	1.35
29			$(10,20)$	92991	95298	2.48
30			$(20,20)$	100673	103216	2.53
31		$(0.5N^s, 0.5R^s_{ij})$	$(1,1)$	38090	39847	4.61
32			$(3,5)$	45218	46124	2.00
33			$(5,5)$	47211	48098	1.88
34			$(5,10)$	50117	51048	1.86
35			$(10,20)$	56673	58208	2.71
36			$(20,20)$	59772	63602	6.41
Average						**3.87**

not only trips to hazmat facilities, but also all other general hazmat trips. Therefore, incorporating both types of hazmat trips within a single modeling framework is a valuable research direction.

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Appendix

A Single-Level Robust Facility Location Problem

$$
\begin{aligned}
\underset{\mathbf{y}}{\text{minimize}} \; & \Big[w_1 \sum_{i\in\mathcal{M}} F_i y_i + w_2 \big(\sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} N^s R^s_{ij} x^s_{ij} + \sum_{s\in S} \rho^s + \sum_{(i,j)\in\mathcal{A}} \xi_{ij} + \Gamma_u \theta_u + \Gamma_v \theta_v \big) \Big] \\
\text{subject to} \; & \sum_{(i,j)\in\mathcal{A}} x^s_{ij} - \sum_{(j,i)\in\mathcal{A}} x^s_{ji} \begin{cases} = 1 & \text{if } i = o(s) \\ \geq -y_i & \text{if } i \in \mathcal{M} \\ = 0 & \text{otherwise} \end{cases} && \forall i \in \mathcal{N}, s \in \mathcal{S} \\
& c_{ij} - \zeta^s_i + \zeta^s_j - \phi^s_{ij} = 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \phi^s_{ij} \leq M(1 - x^s_{ij}) && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \zeta^s_i \leq M[1 - (\sum_{(i,j)\in\mathcal{A}} x^s_{ij} - \sum_{(j,i)\in\mathcal{A}} x^s_{ji} + y_i)] && \forall i \in \mathcal{M}, s \in \mathcal{S} \\
& \rho^s - \sum_{(i,j)\in\mathcal{A}} \eta^s_{ij} + \theta_u \geq \sum_{(i,j)\in\mathcal{A}} K^s R^s_{ij} x^s_{ij} && \forall s \in \mathcal{S} \\
& \xi_{ij} - \sum_{s\in\mathcal{S}} \pi^s_{ij} + \theta_v \geq \sum_{s\in\mathcal{S}} N^s Q^s_{ij} x^s_{ij} && \forall (i,j) \in \mathcal{A} \\
& \eta^s_{ij} + \pi^s_{ij} \geq K^s Q^s_{ij} x^s_{ij} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& x^s_{ij} \in \{0,1\} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& y_i \in \{0,1\} && \forall i \in \mathcal{M} \\
& \zeta^s_i \geq 0 && \forall i \in \mathcal{M}, s \in \mathcal{S} \\
& \zeta^s_i \text{ free} && \forall i \notin \mathcal{M}, s \in \mathcal{S} \\
& \phi^s_{ij}, \rho^s, \xi_{ij}, \eta^s_{ij}, \pi^s_{ij}, \theta_u, \theta_v \geq 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}
\end{aligned}
$$

B Single-Level Robust Network Design Problem

$$
\begin{aligned}
\underset{\mathbf{x},\mathbf{z}}{\text{minimize}} \quad & \Big[\sum_{(i,j)\in\mathcal{A}} \sum_{s\in\mathcal{S}} N^s R^s_{ij} x^s_{ij} + \sum_{s\in S} \rho^s + \sum_{(i,j)\in\mathcal{A}} \xi_{ij} + \Gamma_u \theta_u + \Gamma_v \theta_v \Big] \\
\text{subject to} \quad & \sum_{(i,j)\in\mathcal{A}} x^s_{ij} - \sum_{(j,i)\in\mathcal{A}} x^s_{ji} \begin{cases} = 1 & \text{if } i = o(s) \\ \geq -1 & \text{if } i \in \mathcal{K} \\ = 0 & \text{otherwise} \end{cases} && \forall i \in \mathcal{N}, s \in \mathcal{S} \\
& x^s_{ij} \leq z_{ij} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& c_{ij} - \zeta^s_i + \zeta^s_j + \mu^s_{ij} - \phi^s_{ij} = 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \phi^s_{ij} \leq M(1 - x^s_{ij}) && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \zeta^s_i \leq M[1 - (\sum_{(i,j)\in\mathcal{A}} x^s_{ij} - \sum_{(j,i)\in\mathcal{A}} x^s_{ji} + 1)] && \forall i \in \mathcal{K}, s \in \mathcal{S} \\
& \mu^s_{ij} \leq M[1 - (-x^s_{ij} + z_{ij})] && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& \rho^s - \sum_{(i,j)\in\mathcal{A}} \eta^s_{ij} + \theta_u \geq \sum_{(i,j)\in\mathcal{A}} K^s R^s_{ij} x^s_{ij} && \forall s \in \mathcal{S} \\
& \xi_{ij} - \sum_{s\in\mathcal{S}} \pi^s_{ij} + \theta_v \geq \sum_{s\in\mathcal{S}} N^s Q^s_{ij} x^s_{ij} && \forall (i,j) \in \mathcal{A} \\
& \eta^s_{ij} + \pi^s_{ij} \geq K^s Q^s_{ij} x^s_{ij} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& x^s_{ij} \in \{0,1\} && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S} \\
& z_{ij} \in \{0,1\} && \forall (i,j) \in \mathcal{A} \\
& \zeta^s_i \geq 0 && \forall i \in \mathcal{K}, s \in \mathcal{S} \\
& \zeta^s_i \text{ free} && \forall i \notin \mathcal{K}, s \in \mathcal{S} \\
& \mu^s_{ij}, \phi^s_{ij}, \rho^s, \xi_{ij}, \eta^s_{ij}, \pi^s_{ij}, \theta_u, \theta_v \geq 0 && \forall (i,j) \in \mathcal{A}, s \in \mathcal{S}
\end{aligned}
$$