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A branch-and-price algorithm for emergency humanitarian logistics with a mixed truck-drone fleet

Abhay Sobhanan[1], Sasan Mahmoudinazlou[1], Hadi Charkhgard[1], and Changhyun Kwon[2]

[1]**Department of Industrial and Management Systems Engineering, University of South Florida, Tampa, Florida 33620, USA**
[2]**Department of Industrial and Systems Engineering, KAIST, Daejeon, 34141, Republic of Korea**

Abstract

Humanitarian aid distribution often prioritizes rapid relief operations or emergency services under time constraints, as opposed to commercial transportation problems, where the primary objective is to minimize operational costs. Drones can offer immense potential to achieve this goal by leveraging their aerial mobility. Specifically, drones can surpass ground transportation and navigate directly through disrupted or inaccessible roads, ensuring the quickest path to deliver aid where the ground vehicle may face obstacles. However, drones have limitations in terms of flying range and load capacity. To effectively provide time-sensitive emergency services, combining a ground vehicle with one or more aerial vehicles enhances coverage. Our approach integrates a truck as a mobile depot for multiple drones, where a drone battery is replenished on landing after a flight, and the fleet operates in tandem to serve the locations visited. We formulate a mixed-integer linear programming (MILP) model to maximize the weighted sum of locations served by this mixed truck-drone fleet under time constraints. We further develop a branch-and-price algorithm to solve this problem, where the pricing subproblem is solved using dynamic programming recursions with dominance rules. Our results demonstrate the computational superiority of this method compared to a commercial optimization solver and its potential for expediting aid distribution during an emergency.

Keywords
Traveling salesman, drone, branch-and-price, exact method, humanitarian aid

1. Introduction

Emergency services play a crucial role in various scenarios such as natural disasters, severe weather conditions, and lockdowns due to political, healthcare, security, or public safety reasons. These services cover various necessities, including dietary or medicinal requirements and basic life support products. Such services or surveillance in emergencies must be efficiently executed across the affected locations within a specific time window. In this study, we focus on the last-mile routing of a service vehicle, referred to as a truck, during an emergency. Our goal is to maximize the number of locations served, rather than the conventional optimization of travel distance or routing costs which are commonly addressed in transportation problems.

In recent years, the Traveling Salesman Problem with Drone (TSP-D) [1, 2] has gained significant attention for its potential to reduce the makespan of a truck with the assistance of a drone. In addition, the response team often faces prolonged road travel times due to blocked roads, traffic, accidents or closed/restricted paths during a humanitarian crisis. Aerial vehicles can overcome many of these obstacles and are typically faster than the ground vehicle. Recognizing the advantage of drones in reducing the total travel time, we incorporate truck routing with multiple drones into our problem. Most commercial drones have a load limit of one package per trip. Due to this capacity limit and flying range restriction, utilizing multiple drones in conjunction with a truck can significantly enhance emergency service. We assume that the truck acts as a moving depot for the drones, waiting at a location while the drones take off, serve, and land back on the truck. Moreover, the truck provides service to any location it visits.

Throughout this manuscript, we will refer to the presented transportation problem as the Emergency Traveling Salesman Problem with Multiple Drones (E-TSPMD). The rest of this paper is organized as follows: Section 2 briefly discusses the related literature on truck-drone fleets. In Section 3, we formulate a mixed-integer linear programming (MILP) model to address E-TSPMD and obtain the optimal solutions using an exact branch-and-price (BP) method in Section 4. Finally, our numerical experiments are detailed in Section 5, followed by the conclusions in Section 6.

2. Related Literature

With a remarkable increase in the utilization of aerial unmanned vehicle-assisted deliveries, truck routing with drones has been increasingly studied in recent years. This problem was introduced as the Flying Sidekick TSP (FSTSP) [1], with two different MILP formulations and heuristics employed to obtain solutions. A related variation of this problem, known as TSP-D [2], emerged subsequently. For a comprehensive review of collaborative routing problems involving trucks and drones, one can refer to [3, 4]. Bouman et al. [5] improved the solution quality and computational time for TSP-D by introducing dynamic programming (DP) and greedy approach-based heuristics, respectively. More efficient MILP formulations and a BP algorithm were later employed in [6] to solve TSP-D instances involving up to 39 nodes, to optimality. The authors incorporated a DP approach to address the pricing problem. Among heuristic approaches, the genetic algorithm proposed in [7] stands out with the best-known results for large instances of both TSP-D and FSTSP. Vehicle Routing Problem with Drones (VRP-D), a more generalized problem, is solved in [8] using a BP method, where the pricing problem is initiated with a heuristic column generation for computational advantages.

Despite the extensive research on TSP-D and FSTSP, there exists a limited number of studies that address humanitarian concerns through the integration of mixed truck-drone fleets. Given the immense potential for drone applications in humanitarian logistics, as discussed in [9], it is imperative to conduct additional research on collaborative relief services using trucks and drones. As highlighted in Section 1, employing a mixed truck-drone fleet in emergency services offers significant advantages. Recently, a robust Vehicle Routing Problem with Drones (VRP-D) specifically designed for humanitarian logistics, was tackled using a branch-and-cut algorithm incorporating bidirectional labeling to solve the pricing problem [10]. The majority of related research focuses on minimizing either cost or make-span as the optimization objective. In a different approach, Zhang et al. [11] proposed a collaborative truck-and-drone routing model for humanitarian relief, where the model aims to maximize the value of information collected from the network. The authors obtain the solutions using a column generation framework and discuss a real-world case study. Moreover, many existing studies assume that all locations must be visited by a vehicle, a scenario that may need to be violated in the context of rapid relief logistics under strict time constraints. Our E-TSPMD model addresses this limitation by serving the most number of locations, subject to priority weights, under time constraints during humanitarian emergencies. In our research, we adopt the exact method proposed in [6] with several modifications tailored to our specific problem. We incorporate suitable DP recursions and introduce novel dominance rules to solve the E-TSPMD effectively.

3. Problem Description

In this section, we formulate the E-TSPMD as an MILP to maximize the weighted sum of locations serviced within the specified time constraints.

3.1 Assumptions

Consider a truck that starts from a depot, which acts as a moving depot for l homogeneous drones. Our E-TSPMD model has the following assumptions:

- Each drone has a limited flight time due to battery capacity. After a node visit, a drone returns to the truck where its battery is replaced (or fully recharged) instantaneously. The flying time restrictions have been incorporated during the data preprocessing phase.
- While a drone is in flight, the truck remains stationed at the same node, allowing all drones to complete their visits and return before the truck proceeds. Movement of drones occurs only when the truck is stationary at a node. In essence, the truck serves as a mobile depot for the drones.
- Each drone has a load limit of one product. Given the homogeneous and unit-value nature of all demands in this scenario, service times at nodes are neglected.
- Drone speed is equal to or faster than the truck, a valid assumption given the drone's capability to navigate a straight line path above ground with fewer restrictions compared to road transportation.
- Drones can perform multiple trips from a truck node, including the depot while the truck is stationed there.
- E-TSPMD optimizes routing decisions to maximize the weighted coverage of locations under time restrictions, without considering the fleet's final movement back to the depot.

3.2 Notations

The notations utilized in our model, including sets, parameters, and variables, are defined as follows:

Parameters

n	Total number of locations that require service	l	Number of drones
$0, 0'$	Signifies the same depot for the start node and end node of the truck journey, respectively	r	Drone flying time limit
t_{ij}^T	Truck travel time from node i to node j	t_{ij}^D	Drone travel time from node i to node j
t_j	Time limit for service completion at node j	w_j	Priority weight corresponding to location j

Sets

N	Set of service nodes; $N = \{1,2,\dots,n\}$	$\bar{N}$	Set of all network nodes; $\bar{N} = \{0\} \cup N_{0'}$
$N_{0'}$	Set of service nodes and depot as end node; $N_{0'} = N \cup \{0'\}$	N_0	Set of service nodes and depot as start node; $N_0 = \{0\} \cup N$
D	Set of drone indices $\{1,2,\dots,l\}$	K	Set of drone service indices $\{1,2,\dots,n-l+1\}$, representing the tour count of a drone from a given node

Decision Variables

x_{ij}	Binary decision variables which equals to 1 when the truck serves node j starting from node i, and 0 otherwise	y_{dijk}	Binary decision variable which equals to 1 when drone d starts from truck location i, serves j (and returns to the stationary truck) in k-th order, and 0 otherwise
c_i	Arrival time of the truck at node i		

3.3 Arc-based Formulation

Our E-TSPMD model is formulated as follows:

$$z^* = \max \sum_{i\in N_0}\sum_{j\in N} w_j x_{ij} + \sum_{d\in D}\sum_{i\in N_0}\sum_{j\in N}\sum_{k\in K} w_j y_{dijk} \tag{1}$$

$$\text{s.t.} \sum_{j\in N} x_{0j} \le 1 \tag{2}$$

$$\sum_{i\in N_0} x_{ij} = \sum_{k\in N_{0'}} x_{jk} \qquad \forall j \in N_{0'} \tag{3}$$

$$y_{djk1} \le \sum_{i\in N_0} x_{ij} \qquad \forall j,k \in N; d \in D \tag{4}$$

$$t_{ij}^D y_{dijk} \le r/2 \qquad \forall j,k \in N; d \in D; k \in K \tag{5}$$

$$c_j \le t_j \qquad \forall j \in N \tag{6}$$

$$\sum_{j\in N} y_{dijk} \ge \sum_{j\in N} y_{dijk+1} \qquad \forall i \in N_0; d \in D; k \in K \setminus \{n-l+1\} \tag{7}$$

$$\sum_{j\in N} y_{dijk} \le 1 \qquad \forall i \in N_0; d \in D; k \in K \tag{8}$$

$$\sum_{k\in K} y_{dijk} \le 1 \qquad \forall i \in N_0; j \in N; d \in D \tag{9}$$

$$M(1-x_{ij}) + c_j \ge c_i + t_{ij}^T + 2\sum_{l\in N}\sum_{k\in K} t_{il}^D y_{dilk} \qquad \forall d \in D; i \in N_0; j \in N_{0'} \tag{10}$$

$$M(1-y_{dijk}) + c_j \ge c_i + 2\sum_{l\in N}\sum_{m\in\{1,2,\dots,k-1\}} t_{il}^D y_{dilm} + t_{ij}^D \qquad \forall d \in D; i \in N_0; j \in N_{0'}; k \in K \tag{11}$$

$$\sum_{i\in N_0} x_{ij} + \sum_{i\in N}\sum_{d\in D}\sum_{k\in K} y_{dijk} \le 1 \qquad \forall j \in N \tag{12}$$

$$x_{ii} = 0 \qquad \forall i \in \bar{N} \tag{13}$$

$$x_{ij} \in \{0,1\} \qquad \forall i,j \in \bar{N} \tag{14}$$

$$y_{dijk} \in \{0,1\} \qquad \forall i,j \in \bar{N}; d \in D; k \in K \tag{15}$$

$$c_i \in \mathbb{R}_{\ge 0} \qquad \forall i \in \bar{N} \tag{16}$$

The objective function in equation (1) aims to maximize the priority weights obtained through the traversal of nodes from the set N. Constraints (2) ensure that the truck moves to at most one location from its origin, and (3) indicates the flow conservation constraints for the truck. Constraints (4) specify that a drone movement is only possible from a truck node. The flying range time constraints of the drones are presented in (5). Furthermore, constraints (6) enforce that each potential node visit must be completed within the corresponding time limit. The multi-visit sequence of each drone from a node, where the truck is stationary, is enforced through constraints (7)–(9). The model efficiently employs all available drones concurrently, as required, and thus helps to minimize the overall makespan of the fleet. Since the drones are homogeneous, constraints (7) break the symmetry in the utilization of the drones. Constraints (10) and (11) record the minimal permissible arrival times at each node subject to the propagation of the fleet with respect to time. Constraints (12) and (13) indicate that a node is visited at most once and prevent closed loops in the same location, respectively. Finally, constraints (14)–(16) define the domain of all decision variables.

4. Methodology

Obtaining an optimal solution to the formulation in (1)–(16) becomes computationally challenging with the increase in problem size. To overcome this challenge for medium-sized instances, we propose an exact branch-and-price (BP) algorithm. BP is an iterative procedure that generates the most promising routes dynamically using column generation and pricing in a branch-and-bound tree and finds the best solution that optimizes the problem. The pricing subproblem for E-TSPMD is solved using DP with *ng*-route relaxation. *ng*-route relaxation was proposed in [12] and is proven to be an effective approach in improving the computational speed of the BP method.

4.1 Master Problem

First, we present a set partitioning formulation of E-TSPMD as the master problem with the concept of a set of routes $\mathcal{R}$, that contains all ordered sequences of visits within the problem graph. Consider a binary decision variable λ_r which indicates the selection of a route $r \in \mathcal{R}$. Additionally, let a_{ir} be the number of times node i is visited in path r and m_r be the sum of weights corresponding to node traversals. The set partitioning model is expressed as follows:

$$\max \sum_{r \in \mathcal{R}} m_r \lambda_r \tag{17}$$

$$\sum_{r \in \mathcal{R}} \lambda_r = 1 \tag{18}$$

$$\sum_{r \in \mathcal{R}} a_{ir} \lambda_r \leq 1 \quad \forall i \in N \tag{19}$$

$$\lambda_r \in \{0,1\} \quad \forall r \in \mathcal{R} \tag{20}$$

The master problem (17)–(20) selects a single route that optimizes the objective function in (1) such that each node is visited at most once. The linear programming relaxation of this formulation is referred to as the relaxed master problem (RMP). In the BP procedure, instead of starting with all elements of $\mathcal{R}$ from the beginning, we simply generate them as needed. Specifically, we initialize $\mathcal{R}$ with at least one route and solve the RMP. While the initial route could be a solution with no node traversals ($z^* = 0$), we employ a custom heuristic method to enhance the initial solution. Subsequently, utilizing the dual variables obtained from the RMP solution, the pricing problem is solved to find the most promising route for insertion into $\mathcal{R}$. This iterative process continues until the pricing subproblem fails to generate favorable routes with positive reduced costs for addition. At this stage, the RMP may or may not produce a single route selection due to our subproblem relaxation. If the RMP solution is feasible (integer) to our original problem, we terminate the BP tree. Otherwise, we execute branching on the current solution. One of our branching strategies involves the computation of the total number of times a node is traversed in the fractional solution and selects the node with the most fractional traversal. In addition, we perform branching based on the frequency of node visits, i.e., the total number of times a_{ir} is positive for a given node i and each $r \in \mathcal{R}$ such that $\lambda_r = 1$.

4.2 Pricing Subproblem

We use dynamic programming recursions to solve the pricing subproblem; however, it is noteworthy that the pricing problem is nearly as challenging as the original problem. To alleviate the computational complexity, we add a relaxation to the subproblem, which allows multiple visits to a location under the condition that, between any two consecutive visits, at least one other location is visited. Formally, we record an *ng*-set for a node that contains the nodes visited in the recent steps, and traversal of such nodes is not permissible in the next step. Let $N_i \subseteq N$ represent the neighborhood of node $i \in N$ defined based on the maximum cardinality of the ng-set, a predetermined parameter. Note that $|ng| = n-1$ indicates there are no repeated visits at any node. To boost the BP computational speed, dominance rules are proposed in Proposition 1 and we utilize a simple heuristic for initial column generation at each node.

Let (ng, t, i^T, τ) represent the state of the system, where i^T represents the current truck node, $ng \subseteq N_{i^T}$ is the set of locations that cannot be visited in the next propagation, t is the truck arrival time at the current node, and $\tau = (\tau_d)_{d \in D}$ is a vector where τ_d records the total time drone d is on flight independent of the truck from node i^T. Drone flight time vector values are reset to zeros at the arrival of the truck at a node. Assume u_0 and $u_i (\forall i \in N)$ as the dual variables associated with constraints (18) and (19) for the RMP, respectively. Our labeling algorithm finds a path with the maximum positive reduced cost, i.e., $\max_j \{-u_0 - \sum_{i=1}^{n} u_i a_{ir} + m_j\}$. We initialize a function $f(\phi, 0, 0, \mathbf{0}) = -u_0$ to record the reduced cost and perform the label propagation from a given state as follows:

1. **Add a truck arc**: Given $f(ng, t, i^T, \tau)$, the truck is propagated to each location $j \in N \backslash \{ng \cup \{i^T\}\}$ and the reduced cost is calculated as $f(N_j \cap (ng \cup \{i^T\}), t + \max_{d \in D} \tau_d + t^T_{i^T j}, j, \mathbf{0}) = f(ng, t, i^T, (t_1, \dots, t_l)) - u_j + w_j$.
2. **Add a drone arc**: Propagate a drone d with minimum τ_d to $j \in D_{i^T} \backslash (ng \cup \{i^T\})$, where D_{i^T} is the set of drone eligible nodes from i^T based on the flying range restriction. Calculate the reduced cost as $f(N_{i^T} \cap (ng \cup \{j\}), t, i^T, (\tau_1, \dots, \tau_{\min} + 2t^D_{i^T j}, \dots, \tau_l)) = f(ng, t, i^T, (\tau_1, \dots, \tau_{\min}, \dots, \tau_l)) - u_j + w_j$, where $\tau_{\min} = \min_{d \in D} \tau_d$.

Proposition 1. *Let $f^1 = f(ng^1, t^1, i^T, \tau^1)$ and $f^2 = f(ng^2, t^2, i^T, \tau^2)$. For notational simplicity, we assume τ^1 and τ^2 are sorted in ascending order. $f(ng^1, t^1, i^T, \tau^1)$ dominates $f(ng^2, t^2, i^T, \tau^2)$ if one of the following conditions hold.*

1. *(a) $f^1 > f^2$, (b) $t^1 < t^2$, (c) $\tau^1_l < \tau^2_l$ (d) $ng^1 \subseteq ng^2$*
2. *(a) $f^1 \geq f^2$, (b) $t^1 \leq t^2$, (c) $\tau^1_d \leq \tau^2_d, \forall d \in D$ (d) $ng^1 \subseteq ng^2$, and either*
   (i) $f^1 > f^2$, or (ii) $t^1 < t^2$, or (iii) $\tau^1_d < \tau^2_d$, for any $d \in D$, or (iv) $ng^1 \subset ng^2$.
3. *$f^2 + (n - \Delta(f^2))(\max_{i \in N} w_i) < f^1$, where $\Delta(f^2)$ denotes the step size or number of locations served by f^2.*

Proof. The proof for each of the three dominance rules is straightforward. f^1 dominates f^2 due to its more efficient resource utilization. For the extensions of f^1 and f^2, the objective value of the pricing subproblem for f^1 is at least as favorable as that of f^2. □

5. Results

We consider a grid of size 100 x 100 km^2 for the instance generation process. The node coordinates for the depot and service locations are generated as random integers within the grid, and travel times are calculated based on the Euclidean distance between nodes and the respective speeds of vehicles. Specifically, we set the speeds for the truck and drones at 40 and 60 kilometers per hour, respectively. For a given problem size n, we randomly select the number of homogeneous drones $l \in \{2, 3, 4\}$. To simulate real-world challenges during a calamity, we assume road blockages with a probability of 0.2 and impose a substantial penalty on truck travel times across such obstructed roads. Priority weights assigned to service nodes are randomly generated from the range $[0, 1]$. Our algorithm is implemented in Julia

Table 1: A summary of results for BP in comparison to a commercial optimization solver

	Exact Solver		BP		Gap
Problem Size	Obj	Time (sec)	Obj	Time (sec)	(%)
8	2.50	0.42	2.50	0.22	0.00
9	3.55	3.23	3.55	0.68	0.00
10	3.39	7.09	3.39	2.48	0.00
11	3.44	7.40	3.44	0.28	0.00
12	3.97	35.76	3.97	32.73	0.00
13	4.53	38.01	4.57	21.28	-0.88
14	4.43	209.45	4.45	77.56	-0.45
15	4.67	185.13	4.75	18.81	-1.68
Average	3.81	60.81	3.83	19.26	-0.52

1.7 and computational experiments have been performed on a computer running Windows 10 with Intel Core i7-10700 CPU @ 2.90 GHz processor and 16GB RAM. For the benchmark evaluation of the generated instances, we solve the E-TSPMD formulation using Gurobi optimizer 11.0 with a time limit of 600 seconds for each instance.

Table 1 provides an overview of the average numerical results obtained from 16 instances of each problem size. Throughout our experiments, we consistently set $|ng| = 5$. A negative gap in the results signifies the superior performance of our method compared to the solver under the specified time limit. Notably, the table reveals the exponential growth in computational complexity for the formulation with the increase in problem size, underscoring the substantial computational advantages achieved by our BP method while finding the optimal solutions.

6. Conclusions

In this research, we address the crucial challenge of optimizing humanitarian aid distribution in emergency scenarios using a mixed fleet of a truck and multiple drones. The truck serves as a moving depot for multiple drones, thus enhancing the efficiency of the fleet. Our objective is to maximize the coverage of locations that require service based on priority weights and generate routing decisions for the mixed fleet. To enhance the solution procedure, we devise a tailored branch-and-price method and obtain the optimal routing decisions for medium-sized instances. The numerical results demonstrate the computational superiority of our proposed algorithm over a commercial optimization solver. A potential future improvement in the algorithm lies in the incorporation of cut-generation schemes to enhance our BP method. Furthermore, the operational scenario in an emergency can be improved by considering more vehicles. By using a collaborative system of multiple trucks and drones, the coverage of locations under time restrictions can be improved.

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